Problem Set #1

Due in class Tuesday, October 4.

Part I. Word Problems.

1. Player #1 (the Serf) first chooses either to plant crops (P) or to hide the seed (H). If he chooses P, then at harvest time Player #2 (the Duke) either takes the entire crop (T) resulting in payoffs (0, 3) for the two players, or else shares (S) resulting in payoffs (2, 2). If the Serf chooses H, then the game ends with payoffs (1, 0). Draw the tree (i.e., the extensive form) for this game. (In the next Problem Set you will be asked to solve this game; the assignment now is simply to write it out correctly.)

2. Agressiveland is about to bomb a city in the country of Peaceland. Peaceland has the cities of A and B in its north and the cities of C and D in the south. Agressiveland will send its bombers toward one of the 4 cities in Peaceland. Peaceland with its crude radar will then be able to tell whether the bombers are going north or south, but it will not know which city is targeted. Peaceland will then get to choose which of its four cities to evacuate. If Peaceland evacuates the city that Agressiveland bombs, both countries get a payoff of 0. If the wrong city is evacuated, Agressiveland gets a payoff of 1 and Peaceland a payoff of -1. Write out the extensive form game tree for this game. What is each player’s strategy set?

3. In 1804, sitting Vice President Aaron Burr challenged Alexander Hamilton, former Treasury Secretary, to a gun duel. (H)amilton needs to decide whether to (A)ccept or (R)eject the challenge. Hamilton notifies Burr of his decision by letter. Rejecting the challenge will show Hamilton to be a coward, giving him a payoff of 0 and (B)urr a payoff of 10. At that point the game would end.

If Hamilton (A)ccepts the challenge, the two will meet in Weehawken, New Jersey just across the Hudson river from New York City. The two men will simultaneously decide whether to shoot to (K)ill or shoot the (G)round. If both shoot the ground, they each get a payoff of 5 because both have the honor of having participated in the duel without getting killed. If Hamilton shoots the Ground and Burr shoots to Kill, then Hamilton dies, giving Hamilton a payoff of -1 and Burr a payoff of 2. (Burr’s payoff isn’t higher because committing murder is bad for one’s political career, even in 19th century America!) In the reverse situation, when Burr shoots the Ground and Hamilton shoots to Kill, the payoffs are 2 for Hamilton and -1 for Burr. Finally, if both men shoot to Kill, there’s some probability that one or both men die, so we take the expected payoff to be 1 each in this scenario.

a) Draw the extensive form game tree. Pay attention to what each player knows when he has a decision point.

b) For each player specify his strategy set. (We will not try to analyze this game until later in the quarter.)
4. Two players alternately pick numbers (integers) between 1 and 10, inclusive. The game ends when the cumulative sum equals or exceeds 100.
   
   a. Suppose the last player to name a number is the winner. Which player can force a win, the first mover or the second mover? Specify her winning strategy.
   
   b. Suppose instead that the last player who pushes the sum to 100 or more is the loser. How does that change your answers in part a?

5. Harrington, Chapter 2, exercise 8 (pp. 51)

6. Think of two ideas for project topics that you could suggest to your group. We ask that everyone do this individually. For each idea please briefly explain:
   
   i) Who are the players in the situation that is of interest to you?
   
   ii) Why do these players have differing interests? (e.g. an employee is interested in raising his income and working less while an employer wants to maximize profit, or a suspect is interested in minimizing his expected time in jail, but is less concerned with how long his accomplices are in jail.)

   iii) What are the actions each player can take that have effects both on themselves and others.