Traffic Routing Games -- Intro

- Which route would you choose?
  - The one with the lower delay of course!
Traffic Routing Games -- Intro

- Which route would you choose?
  - Depends on what routes the other cars choose!
  - This is a game.
In this game
- There are many “small” players
  - assume that each player has negligible effect on total traffic
- Payoff functions are the same
  - Delay
Traffic Routing Games -- Intro

- What is the action space of each player?
  - The path taken, i.e. (up, down)

- What is a Strategy Profile?
  - An assignment of route to each car
  - But since, utility functions identical
    - just specify fraction that goes on each route.
    - Call this a “traffic assignment”
What is a Nash Equilibrium of this type of game?
- A traffic assignment such that no car has an incentive to switch paths.
- This is known as a “Wardrop Equilibrium”

For this game: the rates \((\frac{1}{2}, \frac{1}{2})\) specify the Wardrop Equilibrium
Traffic Routing Games -- Intro

- For a traffic assignment to be a Wardrop Equilibrium
  - Traffic on used paths must encounter the same delay
    - Otherwise cars would switch paths
  - Any unused path must have a delay greater than the used paths
What is the Wardrop equilibrium?
  - ($\frac{1}{2}$, $\frac{1}{2}$)

What is the delay the cars face?
  - 1.5
What is the Wardrop equilibrium?
Example

- Is this the Wardrop equilibrium?
  - No
  - Cars can do better
Is this a Wardrop equilibrium?
- No
- Cars can do better
- Red paths have a delay $5/3$, green only $2/3$
- Cars want to switch to green
Is this a Wardrop equilibrium?
- Yes...
- The delay is 2.
- One car switching to red path still finds a delay of 2.
Braess’s Paradox

- But this network had less delay in Wardrop Equilibrium!
- Delay: 1.5

- This network has an additional road
- Delay: 2
- If a social planner could re-route traffic we could achieve a delay of 1.5
Generalizations of Example

- We looked at one particular network
  - With affine latency functions
- And found that the social cost with
  - Selfish routing is 2
  - Social optimal routing is 3/2
- The ratio:

\[
\frac{\text{Total Cost in Nash (Wardrop) Equilibrium}}{\text{Total Cost in Social Optimum}} = \frac{4}{3}
\]

Is sometimes referred to as the “price of anarchy”