Asymmetric n-player games

Civil Unrest \( \sim \) despot coordination failure

500 citizens

Benefit to protesting \( 50x \times m \Rightarrow m \) is \# protesters

Costs personal \( \Rightarrow \) risk being imprisoned, shot in Iran

Radical 100

Progressives 100

Bourgeois 300

\[
\text{Cost} \quad \text{Critical Mass}
\]

\[
\begin{align*}
\text{Radical:} & \quad 6000 \quad 120 \\
\text{Progressive:} & \quad 8000 \quad 160 \\
\text{Bourgeois:} & \quad 20000 \quad 400
\end{align*}
\]

\( \text{property costs consideration} \)

\[
\Pi_r = 50x \times m - 6000 \geq 0 \quad \Rightarrow \quad m_r \geq \frac{6000}{50} \geq 120
\]

\( m = 119.1 \quad \Pi_r = 0 \)

\[
\Pi_p = 50 \times m - 8000 \geq 0 \quad \Rightarrow \quad m_p \geq 160
\]

\[
\Pi_b = 50 \times m - 20000 \geq 0 \quad \Rightarrow \quad m_b \geq 400 \quad \text{so too few radicals are optimal for progressives}
\]

Radical has lower costs so will always rad\_ical protests

Bourgeois prefers when \( \text{same logic}\) radical protests
candidates for a N.E.

1. No one protests
2. Only radicals protest
3. Only radicals and progressives
4. Everyone protests.

Clearly N.E. radical has beliefs no one protests then payoffs are

\[ -6000 \text{ vs } 0 \]

\[ \Downarrow \]

Same logic for progressive, bourgeoisie

All radicals protest \(\Rightarrow\) Not N.E.

\[ T_r = 50 \times 100 - 6000 = -1000 < 0 \]

All radicals and all progressives

\[ T_r = 50 \times 200 - 6000 = 10000 - 6000 = 4000 > 0 \]

\[ T_p = 50 \times 200 - 8000 = 10000 - 8000 = 2000 > 0 \]

1. b

\[ T_b = 50 \times 200 - 20000 = 10050 - 20000 < 0 \]

All

\[ T_b = 50 \times 500 - 20000 = 25000 - 20000 > 0 \]

0

Hence N.E. @ 40% of pop or @ all of pop
Sept 1989  7  16500  $X$
Oct       32  1.4 million  50000*
Nov 28    3.2   "      136000
Dec 21    0.9   "      5943
Jan 1990  26  1.7   "      61000
Feb 1990

Now you can solve 3-player game problems.

Selecting among NEs:

Symmetric games with congestion typically have multiple equilibria.

How do players act on 1st round of play?

No resolution $\Rightarrow$ Empirical test people

Dan LEEPS

<table>
<thead>
<tr>
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<th>R 5</th>
<th>L 1</th>
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<tbody>
<tr>
<td>R</td>
<td>5</td>
<td>1</td>
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<tr>
<td>L</td>
<td>2</td>
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Both prefer R, but L is a NE.

This kind of equilibrium is common in games with tipping or positive PD.

All Windows/All Mac

← →
Symmetric NE

L M H
L 2 3 1 1 0 2
M 1 1 3 3 1 4
H 2 0 4 1 0 0

kind of game symmetric!

L is weakly dominated by H → selection device give preference to undominated strategies

this is not IDSDS
IDWDS

Voting game have 5 NE,
all but one have voters using a weakly dominated strategy
unique undominated NE

shareholder 1 \neq 2 vote option B 
\geq 3 verify it
(3) voting for option C
Payoff dominance Pareto criterion efficiency

A strategy profile satisfies Pareto if there is no other strategy profile for which each player has a strictly higher payoff.

NE individual rationality
payoff dominance collective rationality

by choosing a strategy that is best for everyone.

Payoff dom. in conjunction with NE.

↓

Equilibrium payoff dom.
NE for which no other NE has strictly higher payoff.

Payoff dom. NE

So RR is payoff dom. to LL in Swings.
So Stag is payoff dom. to Hare in Stag Hunt.

Airline Security Game \[ \rightarrow DO \]
Airline Security Game

Not screened in either Malta or Frankfurt

\( n \geq 2 \) airlines, strategy set = \( \{1, 2, 3, 4, 5, 6, 7, 8\} \)

\( s_i \rightarrow \) strategy of airline \( i \)

\( 2^2 \) level of security expenditure

Costs associated with strategy
\( 10 \times s_i \)

Benefits shared by all

Weakest link \( \rightarrow \) overall security

\[ 50 + 20 \times \min \{s_1, \ldots, s_n\} \]

For \( \text{ith} \) airline:

\[ \text{benefit} = 50 + 20 \times \min \{s_1, \ldots, s_n\} - 10 \times s_i \]

Consider \( s_i > \min \{s_1, \ldots, s_n\} \)

Not lowest cost

because

\( \{ \begin{array}{l}
\text{go from } s_i \text{ to } s_i - 1 \\
\text{overall } \min \{s_1, \ldots, s_n\} \text{ unchanged}
\end{array} \) \)

\( 10 \times s_i - 10 \times (s_i - 1) \uparrow \) \( 10 \) more

\( \Rightarrow \) NE all airlines set \( \{\} \) as minimum

No asymmetric equilibria
Need to focus on symmetric strategy 7-9 profiles, each airline chooses same security $s'$ common to all. 

Look at airline one 

$$\Pi_1 = 50 + 20 \times \min \{ s', \ldots, s' \}, \ldots, 5' \cdot 3 - 10 \times s' = 50 + 20s' - 10s'$$

Suppose airline 1 higher security $s'' > s' \Rightarrow \min \{ s'' \geq s', \ldots, 5' \} > s' \Rightarrow s'' > s'$$

$$\Pi_1 = 50 + 20s' - 10s'' = 50 + 10s' - 10(s'' - s') < 50 + 10s'$$

Consider weaker $s < s'$

$$50 + 20 \times \min \{ s, s', \ldots, s' \} - 10 \times s = 50 + 20s - 10s$$

$$= 50 + 10s$$

$$< 50 + 10s'$$

since $s < s'$

all choose $s'$

or any element of $s = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

\[ \begin{array}{cccccc}
2 & 3 & 4 & 5 & 6 & 7 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
10 & 70 & 100 & 170 & 120 & 90 \\
\rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\
60 & 120 & \text{payoff} & \text{dominant} & 7 \text{ N.E. (symmetric)} \\
\end{array} \]

Tex A&M race to bottom 31% highest payoff $\Rightarrow$ 72% lowest payoff
Seven N.E. all airlines choosing most intense 7/10 is highest payoff

Equal Payoff

Logic if players could coordinate on N.E with that property

N.E. credible (all others incredible)

Consider only self-enforcing

Those with N.E.

So pay off dominant ones

Conundrum

Payoff

Weakly dominated

Weakly dominated

Undominated Strategy

Games have societal context

History is common knowledge

⇒ Conspicuous so *must* be common knowledge

London drive left so walk left
Samaritan's Dilemma

1964 residential neighborhood in NYC. Kitty Genovese was brutally murdered. Even though many witnesses heard screams, sociologist behavior of people in big cities changes → diffusion of responsibility.

Game Theory isolate the problem. Everyone cares expresses if police called but incurs cost C so B > C > 0. S = {call, don't call}. 

\[ \Pi_i(c) = B - C > 0 \]
\[ \Pi_i(d) = 0 \] so call! Not NE since not a game but a point of reference.

Case 1

You are only witness

\[ \Pi_i(c) = B - C > 0 \]
\[ \Pi_i(d) = 0 \] so call! Not NE.

Case 2

No other witnesses that know each other and can thus coordinate. One calls gets \[ \Pi_i(c) = B - C \] others get \[ \Pi_i(d) = 0 \].

Case 3

No other witnesses that can't coordinate. N+1 pure strategy NE. Like airline game.

Mixed strategy \[ p \in \{ \text{call}, 1-p \} \]. What is the \( p \) that makes each player indifferent?

\[ c \] pays \( B - C \) for sure.
\[ d \] pays \( B \) with prob \( 1 - (1- p)^N \) \( \frac{0 \text{ prob } (1-p)^N}{(1-p)^N} \)

\[ B - C = B (1 - (1- p)^N) \] solve for \( p \).
\[
\begin{align*}
B \cdot c &= B \cdot B(1-p)^N \\
\frac{c}{B} &= (1-p)^N \\
1-p^* &= \frac{c}{B} \\
p^* &= 1 - \frac{c}{B} \quad \text{prob of calling}
\end{align*}
\]

As \( N \to \infty \), \( p \to 0 \).

\[
\begin{align*}
(1-p^*)^{N+1} &= (1-(1-\frac{c}{B}))^{N+1} \\
&= (1-1+\frac{c}{B})^{N+1} \\
&= \left(1-\frac{c}{B}\right)^{N+1} = \frac{c}{B}
\end{align*}
\]

\[
\begin{align*}
\left(1-p^*\right)^{N+1} &= \frac{c}{B} \\
\frac{c}{B} &< 1
\end{align*}
\]

More people lowers prob.

Sad aspect of NE in anonymous game.