Agenda

0. Continue where John left off
   More NE for 2 player games
   RPS

1. Relation of NE to Best Reply (Response)

2. Intro 3-player games
   Share voting game

3. Relationship between PDPS and NE

4. Algorithm

5. NE in n-player symmetric games
   (tipping point / congestion)

6. Sex Ratio:

   Edlund 1999  Economics of sex preference
   Educ.  Sons/100 daughters
   < 1   99
   1-5  104
   6-8  107
   29   108

   After War I  II

   Male Stressed  shift in sex ratio
   Female Stressed  →  Sex Ratio
Rock, paper, scissors

Lisa

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S</td>
<td>1</td>
<td>0</td>
<td>1</td>
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To be a NE
Lisa plays y → Bart's Best Response to y is c → Lisa's best Response to c is y

Start  Bart

L: R - P - S

There is no NE for RPS.

Sum to zero → Game of Pure Conflict
Constant sum of zero
Sum game

The driving game is a game of mutual interest

Chicken & Telephone lie between conflict & mutual interest

Best Reply (Response)

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
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<tbody>
<tr>
<td>x</td>
<td>10</td>
<td>31</td>
</tr>
<tr>
<td>b</td>
<td>32</td>
<td>02</td>
</tr>
<tr>
<td>c</td>
<td>21</td>
<td>12</td>
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Both Player's payoff & circled (b, x) and (a, y) are NE
All NE satisfy IDSDS
But not all IDSDS are NE
RPS each can be eliminated
but there is NO NE
in stage all survive IDSDS
but only 2 NE

Algorithm
1. Do IDSDS
    ↓
    left with NE
2. Then, do NE Analysis

A strategy profile $\{s_1, \ldots, s_n\}$ is a NE if for all $i$:

1. $s_i$ maximizes player $i$'s payoffs, given he believes that player $j$ will use $s_j(i)$ for all $j \neq i$

2. $s_j(i) = s_i$ for all $j \neq i$

$s_i$ is playing a dual role:
in cond. 1 it is $i$'s decision rule
in cond. 2 it is $j$'s accurate conjecture as to what player $i$ will do

NE is a more useful solution concept
Key lessons from Shreve voter game

Consider AAA

why is it optimal to vote for least preferred for $2 \neq 3$

Neither is pivotal why not Case C

Nash requires that each player acting independently of others can do no better
Intro to 3 Player Games

Stay Hunt

Jean-Jacques Rousseau

H3 chooses stay

Ham

H2 chooses H2

H1 chooses S

No NE

Vote Game vs. Truel

Shareholder Preferences

<table>
<thead>
<tr>
<th>Shareholder</th>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>3rd Choice</th>
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<tbody>
<tr>
<td>25%</td>
<td>1</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>35%</td>
<td>2</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>40%</td>
<td>3</td>
<td>C</td>
<td>B</td>
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</tbody>
</table>

Payoff: 2 if preferred choice (1st)
1 if 2nd choice
0 if 3rd choice

Strategic Form

Case A

Player 3

Consider least preferred 2,3
Voted

Case B

Player 3

Case C

Player 3

Play 3 C
Chapter 5 NE in n-player symmetric games vs asymmetric games

1. Tipping: The more that choose Die, the more attractive it becomes.
2. Congestion: The opposite. The more people on a route, the less attractive.

A game is symmetric:
1. When all players have the same strategy set.
2. Payoffs for all players are the same when using the same strategy.
3. If you switch two players' payoffs, switch as well.

n-player symmetric games

Suppose there is an asymmetric NE, there will be n-1 more symmetric NEs.

<table>
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<th>L</th>
<th>M</th>
<th>H</th>
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<tr>
<td>M</td>
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<tr>
<td>H</td>
<td>2/1</td>
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<td>3.3</td>
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Sweetness Table

<table>
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<tr>
<th></th>
<th>Total(m)</th>
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<tbody>
<tr>
<td>Stay</td>
<td>m &lt; (\frac{n}{2})</td>
</tr>
<tr>
<td>Stay</td>
<td>(\frac{n}{2}) &lt; m</td>
</tr>
<tr>
<td>Nothing</td>
<td>m &lt; (\frac{n}{2})</td>
</tr>
<tr>
<td>Nothing</td>
<td>(\frac{n}{2}) &lt; m</td>
</tr>
</tbody>
</table>

Consider m < \(\frac{n}{2}\) — optimal, but what about n-m?

Case @ m < \(\frac{n}{2}\) but also m < \(\frac{n}{2} - 1\) \(\Rightarrow m + 1 < \frac{n}{2}\)

This could do better without affecting.

\(\circ\) Not NE
Case 2: $m < \frac{n}{2}$ again but $\frac{n}{2} - 1 < m < \frac{n}{2}$

\[ m = \frac{(n-1)}{2} \quad \text{if } n \text{ is odd} \]

Changing $m$ would cause all
Smithe $w \times$ to get zero
So damned if she change,
damned if she doesn't
and it is a NE
also NE \((\frac{n+1}{2})\)

**Mac vs PC**

**network effects**

\[ m + w = n \]

\[ \Pi_m = 100 + 10 \times m \]

\[ \Pi_w = 10 \times w \]

Given $n-1$ Mac users \( w \rightarrow 10 \cdot (10 \times 1) \) \( w=1 \)

\[ m + 100 + 10 \times n \]

Game symmetric applies to all players

**all** \( w \)

\[ \Pi_m = 110 \quad \text{1 mac user} \]

\[ \Pi_w = 100 \times n \]

Assume $w_0$ \( 10 \times n > 110 \) or $n \geq 11$ tipping point

\[ 10 \times n \geq 110 \]

FAC.EBOOK

Network

effects
Sex ratio

During 1889

(Darwin Sexual Selection & The Descent of Man (1871))

Fisher 1930 Natural Selection

Edwards 2000 JTB Clarified

Shaw & Mohler (1953) probably unaware of
Dunlap only aware of Fisher?

\[ \text{Genetic contribution to } G_2 \]

Step 1: NX ♀♂ in G_1 and they supply \( \frac{1}{2} \) of the genes to the G_2

Any contribution of 1 male is:

\[ \frac{X}{2} \cdot \frac{1}{NX} \]

For a given progeny of a ♀♂ in ♀:

\[ \frac{nX}{2NX} \]

Females contribute

\[ \frac{n(1-X)}{2N(1-X)} \]

8 ♀♂ contribute of G_1

\[ \frac{n}{2N} \left( \frac{X}{X} + \frac{1-X}{1-X} \right) \]

♂'s share

\[ \frac{C_{m^2}}{4N} \left( \frac{X}{X} + \frac{1-X}{1-X} \right) \]
for a given \( s \) in \( P \)

\[
C_m = \frac{n}{4N} \left( \frac{x}{X} + \frac{1-x}{1-X} \right)
\]

if \( x = 0.5 \)

\[
C_m = \frac{n}{4N} \left( \frac{x}{X} + \frac{1-x}{1-X} \right)
= \frac{n}{4N} \left( \frac{x}{X} + \frac{1-x}{1-X} \right)
= \frac{n}{4N} \left( \frac{0.5 + 0.5}{1 - 0.5} \right)
= \frac{n}{2N} \leq \text{contribution same regardless of his sex ratio}
\]

any sex ratio as fit as any other \( \hat{N} \)

\[\text{after (2)}\]

\[
C_m = \frac{1}{4} \left( \frac{nx}{NX} + \frac{n(1-x)}{N(1-X)} \right)
\]

\[
C_m = \frac{1}{4} \left( \frac{m}{M} + \frac{f}{F} \right)
\]

\[
\text{large relative to } M
\]

then \( C_m \) \uparrow when \( m > f \)
Modify to include survival

\[ P \downarrow s_1 \quad \frac{n_k s_k}{2N X s_k} + \frac{n (1-x)}{N (1-x) s_k} \]

\[ G_1 \quad s_2 \]

as long as \( s_1 = s_2 \),

same formula.
**Intership Game**

\[
\text{JPM} \div \text{LM}
\]

<table>
<thead>
<tr>
<th>JPM Applicants</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>50</td>
<td>75</td>
<td>100</td>
<td>100</td>
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</table>

Congestion games have an interior NE.

Tipping games have exterior NE.