1. Show that $\text{Accept}(\Lambda)$ is undecidable.

2. Show that if $\mathcal{P} = \mathcal{NP}$, a polynomial time algorithm exists that, given a Boolean formula $\phi$, actually produces a satisfying assignment for $\phi$.

3. Show that $\mathcal{P}$ is closed under Kleene star. That is if $L \in \mathcal{P}$ then $L^* \in \mathcal{P}$ as well. Do the same for $\mathcal{NP}$.

4. Show that the Hamiltonian Path problem is NP-complete by reducing the Hamiltonian Cycle problem to it.

5. Give the definition of NP-completeness.

6. Let $L = \{e(T)|x|t\} :$ where $T$ is a deterministic TM that accepts $x$ within $t$ steps}. Show that $L$ is NP-complete.

7. Show that TSP is NPC by reduction for HC to TSP.

8. Show that the following problem is in $\mathcal{P}$:
   k-CLIQUE:
   INSTANCE: An undirected graph $G = (V, E)$.
   QUESTION: Does $G$ of a clique of size $k$, where $k$ is a constant.

9. Show that the following problem is NP-complete.

   - FEEDBACK VERTEX SET
     INSTANCE: Directed Graph $G = (V, A)$, positive integer $K \leq |V|$.
     QUESTION: Is there a subset $V' \subseteq V$ such that $|V'| \leq K$ and such that every directed circuit in $G$ includes at least one vert from $V''$? (Hint: reduce from Vertex Cover)

   - DOMINATING SET
     INSTANCE: Graph $G = (V, E)$, positive integer $K \leq |V|$.
     QUESTION: Is there a subset $V' \subseteq V$ such that $|V'| \leq K$ and such that every vertex $v \in V - V'$ is joined to at least one member of $V'$ by an edge in $E$? (Hint: reduce from Vertex Cover)

10. Give a definition of the Post Correspondence Problem. What is the complexity of this decision problem?
11. A real number $x$ in the interval (0..1) is called *computable* if there is a TM that enumerates the digits of $x$ after the decimal point. Are all reals in (0..1) computable? Give reasons for your answer!

12. Is there an unrestricted grammar generating the following language:

$$SA = \{ e(T) : T \text{ accepts } e(T) \}$$

What is language class generated by unrestricted grammars?

13. Give a definition of the Busy Beaver Function. Somebody claims that this function can be computed with a new kind of computer that uses light to send messages. What is your opinion of this claim?

14. Recall Collatz’s conjecture:

Each natural number is associated with a sequence as follows: if it is even, then divide the number by two and if it is odd, multiply it by three and add one.

Example: 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1 4 2 1 ...

Conjecture: All natural number go to 1.

This conjecture is open!

Show that if the Halting Problem was decidable then this conjecture could be resolved one way or the other.

Hint: Assume the existence of a TM $T$ that halts on $n$ iff $n$ goes to 1. Further assume that $T_H$ decides the halting problem, i.e. $T_H$ on inputs $e(T)$ and $e(n)$ decides whether $n$ goes to 1.

Now sketch a new TM that halts iff there is a number that does not go to one. Put it all together and show how this outline can be used to resolve the conjecture.

15. Show that $\mathcal{P}$ is closed under polynomial time reductions.

16. Show that if an $\mathcal{NP}$-complete problem is in $\mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$. 