Comments on Homework 2

1. (10 pts) In this problem we consider a modified TM model where the allowable moves are \{R, S, G\} where \(G\) means go to the first tape cell (this was called “reset” in class). In class we claimed that for any Turing Machine \(T\) in the book’s style (with head movements \{L, R, S\}) can be simulated by a \(T'\) in this modified TM model by marking the head location and “shifting” the tape on left moves. Of course, we want \(T'\) to accept the same language as \(T\). For this problem you are to fill in some of the details of this simulation. To keep things simple, we will assume that the \(\Gamma_T = \{0, 1\}\) and \(\Gamma_{T'} = \{0, 1, 0', 1', \Delta\}\).

Assume that at some stage in the computation of \(T\) the tape contents are \(y\alpha_1z\) where \(\alpha \in \{0, 1, \Delta\}\) and \(y, z \in \{0, 1, \Delta\}\^*\), and the underline indicates that the head is reading the “1”. This will be represented on the tape of \(T'\) as \(\Delta^*y\alpha_1'z\) with the head located over the first square (here \(\alpha'\) represents the primed version of the symbol \(\alpha\)). Assume that the head of \(T'\) starts on the leftmost square.

First, ignore the states of \(T\). Draw a transition diagram for \(T'\) that simulates the transition \(1/0, L\) of \(T\). In other words, draw a transition diagram for a subroutine that changes the tape of \(T'\) so that it contains a string \(\Delta^*y\alpha_1'z\) with the head located over the first square (here \(\alpha'\) represents the primed version of the symbol \(\alpha\)).

I will try to post a solution to the web. This basically uses a “shift to the right” routine like that used by the book’s insert subroutine.

Second, what special case can arise when \(\alpha = \Delta\)? Describe (in English) how to handle this case. (You may want to define another tape symbol for \(T'\).)

There are two issues with blanks. The first is that if the simulated machine \(T\) writes blanks, then the simulating machine \(T'\) might not know when to stop shifting the tape. This can be taken care of with an end marker or using a new symbol for the “blanks” written by \(T\).

The second issue is that \(T\) might crash by moving off the left end of the tape. If \(T'\) inserts blanks to “shift” the tape right, then the simulation might not crash (but actually accept the string or compute something). A left-end-marker can let \(T'\) detect this problem and do the appropriate thing (i.e. crash).
Third, give an English description of how $T'$ can keep track of the current state of $T$.

_Students came up with two ways to do this. One way was to remember the state of $T$ in the finite state control for $T'$. Another way was to write the state of $T$ (in unary) on a second tape, and read it when beginning the simulation of the next step of $T$. _

2. (5 pts) Give a detailed English description of a TM with input alphabet \{1\} that computes the function $f(1^i \Delta 1^j) = 1^{ij}$ (i.e. does multiplication in unary).

_Most groups got this one – it is nested loops where you mark off one of the “1”s in $1^i$ and copy the $i^j$ string to the end (using markers on the $1^j$ string to indicate how much has been copied). You then unmark the $1^j$ string and repeat until all the $1^i$ string has been marked. Then you clean up by deleting the (marked) $1^i$ string and the $1^j$ string, leaving the copies at the front of the tape._

3. (5 pts) Problem 9.46 on page 292. Use $m$ (rather than $n_x$) for the fewest number of moves made by the non-deterministic machine $T_1$ in a computation accepting string $x$.

_Most groups got that approximately $2^m$ computations of $T_1$ will be simulated. However each of these computations takes several steps to simulate (a number of steps proportional to the computations length). If $|x|$ (the length of the input) is large, then copying the input to set up each simulation will dominate the time needed to simulate the partial computation. Thus the best answer would be something like $\sum_{i=0}^{m} 2^i \in \Theta(m2^m)$. The actual constant is something like 2 or 3, and there is a (relatively small) amount of start-up/cleanup work._