Homework 2

To be done in groups of 3 (±1).

3 problems, 20 pts, due Wednesday, January 17.

1. (10 pts) In this problem we consider a modified TM model where the allowable moves are \{R, S, G\} where G means go to the first tape cell (this was called “reset” in class). In class we claimed that for any Turing Machine \(T\) in the book’s style (with head movements \{L, R, S\}) can be simulated by a \(T'\) in this modified TM model by marking the head location and “shifting” the tape on left moves. Of course, we want \(T'\) to accept the same language as \(T\). For this problem you are to fill in some of the details of this simulation. To keep things simple, we will assume that the \(\Gamma_T = \{0, 1\}\) and \(\Gamma_{T'} = \{0, 1, 0', 1', \Delta'\}\).

Assume that at some stage in the computation of \(T\) the tape contents are \(ya_1z\) where \(a \in \{0, 1, \Delta\}\) and \(y, z \in \{0, 1, \Delta\}^*\), and the underline indicates that the head is reading the “1”. This will be represented on the tape of \(T'\) as \(\Delta^*ya_1'z\) where \(\Delta^*\) is a string of “\(\Delta\)”s. Assume that the head of \(T'\) starts on the leftmost square.

First, ignore the states of \(T\). Draw a transition diagram for \(T'\) that simulates the transition \(1/0, L\) of \(T\). In other words, draw a transition diagram for a subroutine that changes the tape of \(T'\) so that it contains a string \(\Delta^*ya_1'z\) with the head located over the first square (here \(a'\) represents the primed version of the symbol \(a\)).

Second, what special case can arise when \(a = \Delta\)? Describe (in English) how to handle this case. (You may want to define another tape symbol for \(T'\).)

Third, give an English description of how \(T'\) can keep track of the current state of \(T\).

2. (5 pts) Give a detailed English description of a TM with input alphabet \{1\} that computes the function \(f(1^i \Delta 1^j) = 1^{ij}\) (i.e. does multiplication in unary).

3. (5 pts) Problem 9.46 on page 292. Use \(m\) (rather than \(n_x\)) for the fewest number of moves made by the non-deterministic machine \(T_1\) in a computation accepting string \(x\).