1. Give a deterministic finite state automaton (FA) that accepts all strings over \{a, b\} that don't contain the substring \textit{aba}.

\textit{15pts} Hint: First construct an FA for all strings that contain \textit{aba}.

\[
\begin{array}{c}
\text{start new} \\
\text{at least one occurrence of } \textit{aba}
\end{array}
\]
2) Show that for any regular language $L$, $L^2$ is regular as well, where $L^2 = \{vw : v \in L \text{ and } w \in L\}$.

1) Let $\alpha$ be reg. expr. for $L$
Then $\alpha \alpha$ generates $L^2$

2) Given an NFA $M$ for accepting $L$

- 2 copies of $M$
- A transitions from final of first to start of second
- Undesignate finals of first & start of second
Minimize the following FA. Show your work (give the table). Show the resulting FA in case the number of states was reduced.

20 pts

\[\begin{array}{c|ccc}
2 & 2 & 2 \\
3 & 2 &   \\
4 & 1 & 1 & 1 \\
\hline
1 & 2 & 3 \\
\end{array}\]

\[2 = 3\]

\[\begin{array}{c}
1 \xrightarrow{\alpha, \beta} 2, 3 \\
\end{array}\]

\[\begin{array}{c}
2, 3 \xrightarrow{\alpha} 4 \\
\end{array}\]

\[\begin{array}{c}
4 \xrightarrow{\beta} 2 \\
\end{array}\]
4) Use the "subset construction method" to convert the following NFA to an FA. Label the FA you produce with the subsets of states of original NFA.
7. Give a pair of distinguishable words w.r.t. the language $L = 0(0 + 11)*1$ and show that the pair is distinguishable.

15 pts

Hint: Choose a pair of short words:

\[ \Lambda \text{ and } 01 \text{ are dist.} \]

Because \[ |\{ \Lambda \epsilon, 01 \epsilon \} \cap L | = 1 \text{ for } \epsilon = \Lambda \]
Use the closure properties of regular languages to show that the following language are not regular:

\[ L = \{ w \in \{a, b\} : \text{the number of } a\text{'s in } w \text{ equals the number of } b\text{'s} \} \]

Hint: What simple non-regular language \( L' \) is \( L \) related to?

Assume \( L \) regular

Then \( L \cap a^* b^* \) regular because

- regular languages are closed under intersection
- \( a^* b^* \) is regular

But \( L \cap a^* b^* = \{ a^n b^n : n \geq 0 \} \)

which is not regular

This is a contradiction and therefore \( L \) is not regular
Show that the language \( L = \{0^i1^j0^k : k \leq i + j \} \) is non-regular using the following version of the Pumping Lemma.

For every regular language \( L \) there is a constant \( N \) such that each word \( x \in L \) of length at least \( N \) can be written as \( uvw \) such that the following holds:

1. \(|uv| \leq N\),
2. \(v\) is not the empty word and
3. for all \( i \geq 0 \), \( uv^iw \in L \).

Hint: Try to pump up or down and check which way you arrive at a word that is not in \( L \).

Assume \( L \) regular. Then PL applies. Let \( N \) be a constant of PL.

Choose \( x = 0^N10^{N+1} \).

Since \( x \in L \), and \((x1) \geq N\),

\( x \) can be rewritten as \( uvw \) s.t. i) - iii) hold

By i) \& ii) \( v = 0^q \) for \( q > 0 \).

By iii) \( uvw = 0^{N-q}10^{N+1} \).

That is, for this word

\[ i = N-q, \]
\[ j = 1, \]
\[ k = N+1. \]

Since \( k > i + j = N+1-q \),

\( uvw \notin L \) and we have a contradiction.