1. Give an NFA that accepts all strings over \( \{a, b\} \) containing two consecutive \( a \)'s or two consecutive \( b \)'s (non-exclusive 'or').
\[
(a+b)^* (aa + bb)(a+b)^*
\]

2. For each state of the following NFA give a regular expression for all the strings that lead to that state.

\[
N
\]

\[
A : (01)^*
\]

\[
B : 0(10)^*
\]

\[
C : 0(10)^*(0+1)^+
\]
4. Minimize the following FA. Show your work. Draw the new FA in case the number of states was reduced.

3. Convert the following λ-NFA into a regular expression. Use the algorithm given in class that eliminates one node at a time and creates arcs that are labeled with regular expressions.

\[
(a^+ + a(a+ba^+)) \cdot (b(a^+ + a(a+ba^+)))^* \]
Use the "subset construction method" to convert the following NFA to an FA. Show the steps of your construction as well as the final FA.
Show that \( L := \{ww : w \in \{a, b\}^*\} \) is not regular by exhibiting an infinite set that is pairwise distinguishable.

Hint: Use \( S = \{ba^i : i \geq 1\} \).

Prove for each pair of words of \( S \) that it is pairwise distinguishable.

**Proof:** Let \( x \) and \( y \) be two arbitrary distinct elements of \( S \). That is,

\[
x = ba^j, \quad y = ba^k, \text{ and } j \neq k, \quad j \geq 1, \quad k \geq 1
\]

Select \( z = ba^j \).

\[
xz = (ba^j)(ba^j) \in L
\]

\[
yz = (ba^k)(ba^j) \notin L, \text{ since } k \neq j.
\]

It follows that the infinite set \( S \) is pairwise distinguishable w.r.t. \( L \).

Thus \( L \) can't be regular since distinguishable words have to end up in different states in a DFA that accepts \( L \).
Show that if a language $L$ over some finite alphabet $\Sigma$ is regular then the language $\overline{L}$ of all suffixes of words in $L$ is also regular.

Formally $\overline{L}$ is defined as $\{w \in \Sigma^* | \exists v \in \Sigma^* \text{ such that } uv \in L\}$. 

Hint: There are many solutions to this problem. One solution uses other similar closure properties of regular sets that were discussed in class.

I) Since $L$ is regular, there is an FA that accepts it. We will construct a $\overline{NFA} \tilde{M}$ that accepts $\overline{L}$. Since $\overline{NFA}$'s accept regular languages, we are done.

To construct $\tilde{M}$ from $M$, add a new state $q_f$ to $M$ and make it the start state of the new machine. Add a transition to all states of $M$ reachable from the old start state of $M$.

This condition is necessary!

II) $\text{SUFF}(L) = \text{REV}(\text{PREV}(\text{REV}(L)))$

$\text{REV}(L) = \{w \in \Sigma^* : w^R \in L\}$

$\text{PREV}(L) = \{w \in \Sigma^* : \exists \ v \in \Sigma^* \text{ and } uvw \in L\}$

We showed in class that regular languages are closed under the operations $\text{REV}$ and $\text{PREV}$, and thus $L$, and are closed under $\text{PREV}$ as well.
Use the Pumping Lemma for regular languages to show that \( \{a^i b^j | 0 \leq 2i \leq j \} \) is not regular.

You can use the following Pumping Lemma:

For every regular language \( L \) there is a constant \( N \) such that each word \( z \in L \) of length at least \( N \) can be written as \( uwv \) such that the following holds:

i) \( |uv| \leq N \),

ii) \( v \) is not the empty word and

iii) for all \( i \geq 0 \), \( uv^i w \in L \).

Assume \( L \) is regular. Then the PL holds for \( L \).

Let \( N \) be the constant of the PL.

Let \( x = a^N b^{2N} \), since \( x \in L \) and \( |x| > N \),

\( x \) can be written as \( uvw \) such that \( i), ii) \) and

\( iii) \) of PL hold.

i) implies that \( uv \in a^* \)

ii) implies that \( v \in a^+ \).

iii) implies that

\[ uv^2 w = a^{N+|uv|} b^{2N} \in L \]

By the definition of \( L \) this means that

\[ 2(N+|uv|) \leq 2N \]

This can't be true since \( |uv| > 1 \).

We have a contradiction to the assumption that \( L \) is regular!