CMPS 130
Final Review Problems

These problems are meant as review of the material covered since the last midterm. Homework assignments 6, 7 and 8, and their solutions, should also be considered as review for this material. Consult the previous review sheets and midterm solutions for review of prior material.

1. Write down productions for CFGs that generate the following languages over \{a, b\}. In each case make up a few strings in the language and give derivations of those strings.
   a. \(\text{NonPal} = \{ x \in \{a, b\}^* \mid x^r \neq x \}\)
   b. \(\text{AeqB} = \{ x \in \{a, b\}^* \mid n_a(x) = n_b(x) \}\)
   c. \(\text{AgtB} = \{ x \in \{a, b\}^* \mid n_a(x) > n_b(x) \}\)

2. Write down productions for regular grammars that generate the following regular languages over \{a, b\}. Notation: \(L(r)\) denotes the regular language corresponding to the regular expression \(r\). Hint: first draw an NFA for the language, eliminate any \(\lambda\)-transitions, then write down a regular grammar.
   a. \(L((a + b)^*abb)\)
   b. \(L((ab^*a + baaba)^*)\)
   c. \(L((ba)^*(bab + aa)^*)\)

3. Write productions for a CFG that generates the language \(L = \{ a^i b^j \mid 2i = 3j \}\). Hint: look at hw7 solutions problem 4.10c.

4. Describe the language (a subset of \{a, b\}*) generated by the CFG with the following productions. Hint: consider hw7 solutions problems 4.1gh.
   \[
   \begin{align*}
   S &\rightarrow aT \mid bT \\
   T &\rightarrow aU \mid bU \mid \lambda \\
   U &\rightarrow aS \mid bS
   \end{align*}
   \]

5. Prove that the union of two CFLs is a CFL. (This is part (1) of Theorem 4.9 whose proof is on p.136 of the text. The proof is also in the lecture notes from 5-24-16 and 5-26-16.)

6. Let \(\mathcal{F}\) denote the set of finite languages and \(\mathcal{R}\) the set of regular languages over an alphabet \(\Sigma\).
   a. Prove that every \(L \in \mathcal{F}\) is a CFL. (Use the recursive definition of \(\mathcal{F}\), structural induction and Theorem 4.9.)
   b. Prove that every \(L \in \mathcal{R}\) is a CFL. (Use the recursive definition of \(\mathcal{R}\), structural induction and Theorem 4.9. This is also problem 4.21 in the text.)

7. For each of the DFAs pictured in figure 2.44 (p.79 of the text), write down a regular grammar that generates the language accepted by the DFA.

8. Draw a PDA that accepts the language of even length palindromes over \{a, b\}.

9. State the Pumping Lemma for CFLs.

10. Prove that the language \(L = \{ x \in \{a, b, c\}^* \mid n_a(x) = n_b(x) = n_c(x) \}\) is not context free.
11. Problem 6.2a on p. 220 of the text. (This is the pumping for CFLs lemma again.)

The following problems may or may not be included depending on how far we get on the last day of class, Thursday 6-3-16.

12. Draw a transition diagram for a Turing Machine accepting the language corresponding to the regular expression: \((a + b)^*aba(a + b)^*\).

13. Trace the TM in Figure 7.5 of the text (p. 232), accepting the language \(\{ xx \mid x \in \{ a, b \}^* \} \) on the input strings \( aa \) and \( ab \). Show the machine configuration at each step.