The “Problems to be turned in” are to be done in groups of 3 (or 2 if necessary), each group should turn in one set of solutions with each group member’s name and e-mail account clearly indicated at the top. Students are to select their own groups and review the homework policies on the course information sheet.

Note: There will also be an individual Gradiance homework for this week.

Problems to be turned in (15 pts):

1. (3 pts) Exercise 8.4.1 on page 349 - Describe multitape TMs for languages $L_1 = \text{all strings having an equal number of 0's and 1's over } \Sigma = \{0, 1\}$; $L_2 = \{a^n b^n c^n | n \geq 1\}$ over $\Sigma = \{a, b, c\}$; and $L_3 = \{ww^R | w \in \{0, 1\}^*\}$ over $\Sigma = \{0, 1\}$.

2. (2 pts) Exercise 8.4.2 part b) only.

3. (2 pts) Exercise 9.1.2 (write code for TM)

4. (4 pts) Assume that we have three languages, $L_1$, $L_2$, and $L_3$, over $\{0, 1\}$ such that:
   - Each of $L_1$, $L_2$, and $L_3$ is recursively enumerable.
   - $L_1 \cup L_2 \cup L_3 = \{0, 1\}^*$.
   - The languages are (pairwise) disjoint, i.e. $L_1 \cap L_2 = L_2 \cap L_3 = L_1 \cap L_3 = \emptyset$.

   Show that language $L_1$ is actually recursive.

5. (4 pts) Suppose the $L$ is a recursively enumerable language, but that $L$ is not recursive. Let $M$ be a Turing Machine $M$ that accepts $L$ (i.e. for all $w \in L$, $M$ enters a final state on input $w$, and for all $w \notin L$, machine $M$ either halts in a non-final state or runs forever). Prove that infinitely many inputs must cause $M$ to run forever. Hint: Assume to the contrary that $M$ runs forever on only finitely many inputs, and find a contradiction to the fact that $L$ is not recursive.