CMPS 130 Written Homework 1

Due Monday October 4

Problems 1-3 are to be done in groups of 3 (or 2 if necessary), each group turning in a single set of solutions with each student’s name and e-mail account clearly indicated at the top. Students are to select their own groups and review the homework policies on the course information sheet.

When I say/write “show” it means prove, and when I say/write “prove” it means ”prove formally”.

Note: There is also a Gradiance homework due on Monday the 4th.

3 Problems to be turned in, one to be done in eCommons, 20 pts

1. (4 pts) Use mathematical induction to prove that \( \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \) for all \( n \geq 1 \).

2. (5 pts) Consider the function \( f(n) \) defined on the positive integers as follows:
   - \( f(1) = 1 \)
   - for \( n \geq 1 \), \( f(2n) = f(2n + 1) = 2f(n) \).

   Use mathematical (strong) induction to prove that for every positive integer \( n \), \( f(n) \) is the largest power of 2 less than or equal to \( n \).

3. (4 pts) For each of the following languages over the alphabet \( \Sigma = \{0, 1\} \) give a DFA (as a transition diagram) that recognizes the language.
   (a) The language consisting of all strings that start with a 1 and end with a 0.
   (b) The language consisting of all strings that contain 111 as a substring.
   (c) The language consisting of all strings that do not contain 111 as a substring.
   (d) The language that contains both the string ”0” as well as all (binary) strings that start with a 1, but contains neither the empty string nor any other string starting with 0. (This language consists of the standard binary representations of the non-negative integers without leading 0’s.)

4. (2 pts) Each student should post one message in the Pictures discussion forum on the class eCommons page that contains a picture from which they can be identified (preferably portrait-style, to help us match names and faces).