1. (10 Points) Prove that if \( f(n) = \Theta(g(n)) \), then \( f(n)^2 = \Theta(g(n)^2) \).

2. (10 Points) Let \( f(n) \) be a positive increasing function and \( c \) a positive constant. Is it necessarily true that \( f(cn) = \Theta(f(n)) \)? Either prove this or give a counter-example.

3. (10 Points) Let \( f(n) \) and \( g(n) \) be asymptotically positive functions. Prove that

\[
f(n) + g(n) = \Theta(\max(f(n), g(n)))
\]

4. (20 Points) List the following functions from lowest to highest asymptotic order. Indicate whether any two (or more) are of the same asymptotic order. Justify your answers.

\[
2^{\lg(n)}, (\lg n)^{\lg n}, n^{1/\lg n}, \ln(\ln(n)), 4^{\lg n}, 2^{(\lg n)^2}, \sqrt{2}^{\lg n}, n, \sqrt{2n}, 1, \lg(n!)
\]

5. (20 Points) Let \( f(n) \) and \( g(n) \) be positive functions. Prove or disprove each of the following.

   a. (10 Points) If \( f(n) = \Theta(g(n)) \) then \( 2^{f(n)} = \Theta(2^{g(n)}) \).
   
   b. (10 Points) If \( f(n) = \Theta(g(n)) \) then \( \lg(f(n)) = \Theta(\lg(g(n))) \). Assume here that \( f(n) \geq 2 \) and \( g(n) \geq 2 \) for all sufficiently large \( n \).

6. (20 Points) Determine the asymptotic order of each of the following expressions, i.e. for each expression, find a simple function \( g(n) \) such that the expression is in the class \( \Theta(g(n)) \). Prove your answers.

   a. (10 Points) \( \sum_{i=1}^{n} \log(i) \)

   b. (10 Points) \( \sum_{i=1}^{n} a^i \) where \( a > 0 \) is a constant. (Hint: consider the cases \( 0 < a < 1 \), \( a = 1 \), and \( a > 1 \) separately.)