continuing...

Some algebra...

\[
\overline{t}(n) = (c_4 + c_5 + c_6) \left( \sum_{j=2}^{n-1} t_j \right) \\
+ (c_1 + c_2 + c_3 - c_5 - c_6 + c_7) n \\
+ (-c_2 - c_3 + c_5 + c_6 - c_7)
\]

Cases: Best, Worst, Average

Best: \( t_j = 1 \)

\[
\sum_{j=2}^{n-1} t_j = (n-1)
\]

\[
\overline{t}(n) = (c_1 + c_2 + c_3 + c_4 + c_7)n + (-c_2 - c_3 - c_4 - c_7)
\]
Worst: \( t_j = i \)

\[
\sum_{j=2}^{n} t_j = \left( \sum_{j=1}^{n} j \right) - 1 = \frac{n(n+1)}{2} - 1
\]

\[
\bar{1}(n) = (\quad ) n^2 + (\quad ) n + (\quad )
\]

Average: \( t_j = \frac{i}{2} \)

\[
\sum_{j=2}^{n} t_j = \frac{1}{2} \left( \frac{n(n+1)}{2} - 1 \right)
\]

\[
\bar{1}(n) = (\quad ) n^2 + (\quad ) n + (\quad )
\]
<table>
<thead>
<tr>
<th>Scenario</th>
<th>$T(n)$</th>
<th>Asymptotic Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>$an+b$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Worst</td>
<td>$cn^2+dn+e$</td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td>Avg</td>
<td>$ln^2+gn+nh$</td>
<td>$\Theta(n^2)$</td>
</tr>
</tbody>
</table>

Where $a-h$ depend on Comp. device.
Ex. Suppose we have 4 algorithms A, B, C, D all solving the same problem, with (worst case) run times:

<table>
<thead>
<tr>
<th></th>
<th>Run time</th>
<th>Asym. Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$n^2$</td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td>B</td>
<td>$10n^2$</td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td>C</td>
<td>$10n^2 + 2n + 100$</td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td>D</td>
<td>$1000n + 10800$</td>
<td>$\Theta(n)$</td>
</tr>
</tbody>
</table>

Diagram showing the complexity relationship between the algorithms.
\[ \frac{C}{R} = \frac{10n^2 + 2n + 100}{10n^2} \]

\[ = 1 + \frac{1}{5n} + \frac{10}{n^2} \]

\[ \downarrow \quad \downarrow \quad \text{as } n \to \infty \]

\[ 0 \quad 0 \]

**comp. R to C**

**comp. A to B**

*equalize by running B on a faster machine.*
Strategy: Pick a basic operation (barometer or \( \cdot \)) and count # executions in best, worst, avg. cases.

Sorting algorithm:

basic operation is a comparison of array elements.

In insertion sort,

line 4: \[ \text{temp} < A_i \]
We set

<table>
<thead>
<tr>
<th></th>
<th># comparisons</th>
<th>Asymptotic growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>best</td>
<td>$n-1$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>worst</td>
<td>$\frac{n(n-1)}{2}$</td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td>avg.</td>
<td>$\frac{n(n-1)}{4}$</td>
<td>$\Theta(n^2)$</td>
</tr>
</tbody>
</table>

\[ \text{worst} = 1 + 2 + 3 + \ldots + (n-1) \]
\[ = \frac{n(n-1)}{2} \]
Merge Sort:

Divide & Conquer:

\[ I_1, I_2, \ldots, I_k \]

\[ S_1, S_2, \ldots, S_k \]

\[ (\text{Problem instance}) \]

\[ (\text{sub-instances}) \]

\[ (\text{sub-solutions}) \]

\[ (\text{Problem solution}) \]
\texttt{MergeSort}(A, p, r)

1.) \textbf{if} \ p < r \\
2.) \quad q = \lfloor \frac{p + r}{2} \rfloor \\
3.) \quad \texttt{MergeSort}(A, p, q) \\
4.) \quad \texttt{MergeSort}(A, q + 1, r) \\
5.) \quad \texttt{Merge}(A, p, q, r)