QUIZ 1
CMPS 101 - Winter 02
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Name: __________________________
Student ID: ______________________

This exam is closed book and closed notes. Show all work. Partial credit given for partial solutions. Presentation counts! Be legible and coherent for full credit.

1. (25 points) Prove

\[
\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}
\]

where \( r \) is a real number and \( r \neq 1 \) and \( r \neq 0 \).

**Hint:** One method is to use induction on \( n \).
2. (25 points) Find the limit and simplify your answer.

\[ \lim_{x \to \infty} \frac{x \cdot \ln x}{x^{1.01}} \]
3. (25 points)

- **Part 1** (12 points)
  Write C or Java code or pseudocode for “power,” an integer exponentiation function with the following specifications:
  
  (a) has the C function declaration
  long int power(int n, int k);
  or the Java prototype
  public long power(int n, int k)
  if you write code, or is a function that takes two integers and returns an integer if you write pseudocode.
  
  (b) power(n, k) returns $n^k$ if both $n$ and $k$ are positive integers.
  Otherwise, power(n, k) returns 0.
• **Part 2** (13 points)
  Write a mathematical expression in terms of \( k \) for the number of multiplications performed by your algorithm for the function \( \text{power}(n, k) \) you created in Part 1 above. If your algorithm is recursive you may give a recurrence and you need not solve the recurrence (solving the recurrence may result in extra credit).
4. (25 points) You have a complete deck of playing cards with 52 cards. The deck of cards is completely shuffled so we can assume that any arrangement of the cards is equally likely. There is only one Queen of Spades in the deck. You turn cards up one at a time until you find the Queen of Spades, then you stop. What is the expected number of cards you will turn up?

**Formula for expected value:**

\[
E(X) = \sum_{u \in \text{domain of } X} u \cdot \Pr(X = u)
\]

**Hint:** First find the probability that the Queen of Spades is in a particular position \(i\) of the 52 positions in the deck. Every position is symmetric to every other position, and **nothing fancy is required!!**

Then, count how many cards you will turn up when the queen is in position \(i\) and apply the formula for the expected value.
5. **Extra Credit**

In class, we gave the formula for the derivative of a product of two functions: \([f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)\). Derive a formula for the derivative of a product of \(n\) functions: \([f_1(x)f_2(x)\ldots f_n(x)]'\). Prove your formula.