QUIZ 1
CMPS 101 - Winter 02
Thomas Raffill

Name: __________________________
Student ID: ______________________

This exam is closed book and closed notes. Show all work. Partial credit given for partial solutions. Presentation counts! Be legible and coherent for full credit.

1. (25 points) Prove

\[ \sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1} \]

where \( r \) is a real number and \( r \neq 1 \) and \( r \neq 0 \).

**Hint:** One method is to use induction on \( n \).

12 points for correct proof of basis. 10 points if proof has minor mistakes. 8 points if you substituted a particular value for \( r \) when trying to do induction on \( n \). 4 points if you correctly stated basis but didn’t prove. 2 points for any kind of basis. We also accepted a basis with \( n = 1 \) instead of \( n = 0 \).

13 points for correct proof of inductive step. 11 points for one minor mistake, 9 points for two minor mistakes. 5 points for correctly stating inductive hypothesis with no proof. 3 points for mentioning any inductive step.

We also accepted a correct non-inductive proof.
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2. (25 points) Find the limit and simplify your answer.

\[ \lim_{x \to \infty} \frac{x \cdot \ln x}{x^{1.01}} \]

25 points for correct solution with correct steps. 20 points for 1 incorrect step. 15 points for 2 incorrect steps. 10 points for 3 or more incorrect steps but having at least a few correct steps. 5 points if instead of calculus you tried the expression for large values and got the right answer.
3. (25 points)

- **Part 1** (12 points)
  Write C or Java code or pseudocode for “power,” an integer exponentiation function with the following specifications:

  (a) has the C function declaration
  
  long int power(int n, int k);

  or the Java prototype
  
  public long power(int n, int k)

  if you write code, or is a function that takes two integers and returns an integer if you write pseudocode.

  (b) power(n, k) returns $n^k$ if both $n$ and $k$ are positive integers.

  Otherwise, power(n, k) returns 0.

5 points for correctly catching the cases $n \leq 0, k \leq 0$. 3 points if you allow $n = 0, k = 0$. 7 points for correctly computing $n^k$ by repeated multiplication. 3 points for any algorithm using multiplication, even if wrong.
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• **Part 2** (13 points)
  Write a mathematical expression in terms of $k$ for the number of multiplications performed by your algorithm for the function $\text{power}(n, k)$ you created in Part 1 above. If your algorithm is recursive you may give a recurrence and you need not solve the recurrence (solving the recurrence may result in extra credit).
  
  Full credit for correctly giving exact count or big-$O$ or big-$\Theta$. 11 points for saying “approximately $k$.” 3 points for any expression with $k$. Extra credit for correctly stating a recurrence for your algorithm.
4. (25 points) You have a complete deck of playing cards with 52 cards. The deck of cards is completely shuffled so we can assume that any arrangement of the cards is equally likely. There is only one Queen of Spades in the deck. You turn cards up one at a time until you find the Queen of Spades, then you stop. What is the expected number of cards you will turn up?

**Formula for expected value:**

\[ \mathcal{E}(X) = \sum_{u \in \text{domain of } X} u \cdot \Pr(X = u) \]

**Hint:** First find the probability that the Queen of Spades is in a particular position \(i\) of the 52 positions in the deck. Every position is symmetric to every other position, and **nothing fancy is required**!! Then, count how many cards you will turn up when the queen is in position \(i\) and apply the formula for the expected value.

7 points for realizing that the probability that the Queen of Spades is in position \(i\) is \(\frac{1}{52}\). 8 points for realizing that when the Queen of Spades is in position \(i\) you will turn up \(i\) cards, 5 points for correctly combining the probability and the number of cards in a formula for the expected value, 5 points for correctly calculating the expected value. Also, 10 points for realizing intuitively that the expected value is about \(\frac{52}{2}\) or \(\frac{52}{2}\) even without the right math to support it.
5. **Extra Credit**

In class, we gave the formula for the derivative of a product of two functions: 
\[ (fg)'(x) = f'(x)g(x) + f(x)g'(x). \]
Derive a formula for the derivative of a product of \( n \) functions: 
\[ (f_1 f_2 \cdots f_n)'(x). \]
Prove your formula.

Excellent for a correct answer. Good try for a creative but not entirely sufficient answer. Incomplete attempt for the correct formula without the proof.