Homework 3
Show all work.

1. Apply the following methods to solve the recurrence, $T(n) = T(n/3) + 2$.
   (a) Transformation with $n = 3^a$
   (b) Iteration
   (c) Tree diagram
   (d) Check your answer by substituting it back into the recurrence

2. Let $f(n)$ be a positive function $f(n) > 0$, and let $k$ be a constant. Prove or give a counter-example for each of the following statements.
   (a) $f(n) + k$ is $\theta(f(n))$.
   (b) $f(n + k)$ is $\theta(f(n))$.
   (c) $f(n)$ is $O(2^{f(n)})$.
   (d) $f(n)$ is $o(f(2^n))$.

3. A complete binary tree is one in which every node except leaf nodes has two descendents. Prove that more than half the nodes in a complete binary tree are leaf nodes.

4. Extra Credit:
   A complete $k$-ary tree is one in which every node except leaf nodes has $k$ descendents. State the strongest result you can about the ratio of the number of leaf nodes to the number of all nodes, and prove your result.