1. Let $T$ be a tree with $n$ vertices and $m$ edges. Prove that $m = n - 1$ by induction on $m$.

2. Let $G$ be an acyclic graph with $n$ vertices, $m$ edges and $k$ connected components. Use the result of the preceding problem to prove that $m = n - k$. (Hint: apply the preceding result to each of the $k$ trees composing $G$.)

3. Use the iteration method to find an exact solution to the recurrence:

$$T(n) = \begin{cases} 
1 & 1 \leq n < 3 \\
2T(\lfloor n/3 \rfloor) + 5 & n \geq 3 
\end{cases}$$

4. Use the iteration method on the following recurrence

$$T(n) = \begin{cases} 
3 & 1 \leq n < 5 \\
4T(\lfloor n/5 \rfloor) + n & n \geq 5 
\end{cases}$$

to show that

$$T(n) = \sum_{i=0}^{\lfloor \log_5(n) \rfloor - 1} 4^i \lfloor \frac{n}{5^i} \rfloor + 3 \cdot 4^{\lfloor \log_5(n) \rfloor}$$

and hence $T(n) = \Theta(n)$. 