Exercise 1:

Area of flowers model:

\[ \text{Area of flowers} \]

\[ \text{growth} \]

\[ \text{fraction occupied} \]

\[ \sim \]

\[ \text{growth multiplier} \]

\[ \text{intrinsic growth rate} \]

\[ \text{actual growth rate} \]

\[ \text{suitable area} \]

Direct calculation using logistic function:

\[ \text{intrinsic growth rate} \]

\[ r \]

\[ A_{\text{zero}} \]

\[ A \text{ of } t \]

\[ K \]

Comparisons

The standard (DT=1.0) shows significant differences between the two graphs.
As the DT value gets smaller, the model becomes closer to the direct formula calculation.
At DT= 0.1, they are almost identical, but still not exact.

At DT=0.01, all differences have disappeared:
Exercise 2:

The disturbance itself can be modeled with three extra converters like this:

```
Area of flowers growth decay fraction occupied ~ growth multiplier actual growth rate

\[
\text{Disturbance Amount} = \text{growth multiplier} \times \text{actual growth rate}
\]

The equations for the whole graph should look like this, the additional changes are highlighted

\[
\text{Area of flowers}(t) = \text{Area of flowers}(t - dt) + (\text{growth} - \text{decay}) \times dt
\]

\[
\text{INIT Area of flowers} = 10
\]

\[
\text{growth} = \text{Area of flowers} \times \text{actual growth rate}
\]

\[
\text{decay} = \text{Area of flowers} \times \text{decay rate} + \text{Disturbance}
\]

\[
\text{A}_{\text{of t}} = \text{Azero} \times \exp(r \times \text{TIME}) / \text{K} \]

\[
\text{actual growth rate} = \text{intrinsic growth rate} \times \text{growth multiplier}
\]

\[
\text{Azero} = 10
\]

\[
\text{decay rate} = 0.2
\]

\[
\text{Disturbance} = \text{IF} (\text{TIME}=\text{Target Year}) \times \text{Disturbance Amount} \text{ ELSE } 0
\]

\[
\text{Disturbance Amount} = \text{Area of flowers} \times 0.2
\]

\[
\text{fraction occupied} = \text{Area of flowers} / \text{suitable area}
\]

\[
\text{intrinsic growth rate} = 1
\]

\[
\text{K} = 800
\]

\[
\text{r} = \text{intrinsic growth rate} - \text{decay rate}
\]

\[
\text{suitable area} = 1000
\]

\[
\text{Target Year} = 15
\]

\[
\text{growth multiplier} = \text{GRAPH} (\text{fraction occupied})
\]

\[
(0.00, 1.00), (0.2, 0.8), (0.4, 0.6), (0.6, 0.4), (0.8, 0.2), (1.00, 0.00)
\]

We added a new term to “decay” flow which was named “Disturbance.” This converter which was populated by an IF statement would produce either 0 or 0.2*Area_of_flowers depending on if TIME=Target_Year.

Note, for this to work, you must have DT=1.0 in your run specs, otherwise, fractional years will mean additional disturbances.

Another way to accomplish this would be to introduce a new outward flow from the stock which only reduces the stock on the 15th year by 20%. You must note, however, that this method will not show the reduction reflected in the decay line on the graph.
This graph shows the disturbance on the 15th year as an initial increase in decay which then lowers the area of flowers. When the area is lowered to the levels comparable to years 7-9, the growth increases because the growth multiplier corresponds to a lower value for “fraction occupied.” The area then continues to increase at a declining rate until it arrives again at the pre-disturbance levels.
Exercise 3:

We need 4 graphs for this problem. The easiest way to accomplish this in STELLA is to simply copy and paste the whole model 3 times, and then change the initial stock amount to 5, 15 and 20 in the new ones respectively.

As the graph shows, the initial amount is actually not a factor in the final area of flowers. The only difference the initial amount makes is how fast the area increases.
Exercise 4:

First, we introduce the disturbance to figure 6.10. The disturbance is defined as 40% additional departure in the sales force on the 15\textsuperscript{th} year. As before, we introduce 3 new converters to accomplish the job. We use an IF statement in the disturbance converter to facilitate the additional departure only on year 15. We also need to make sure that DT=1.0, and length of simulation is from 0 to 30 under run specs.

\begin{align*}
\text{siza of sales force}(t) &= \text{siza of sales force}(t - dt) + (\text{new hires} - \text{departures}) \times dt \\
\text{INIT siza of sales force} &= 50 \\
\text{new hires} &= \text{hiring fraction} \times (\text{budgeted size of sales force} - \text{siza of sales force}) \\
\text{departures} &= \text{siza of sales force} \times \text{exit rate} + \text{disturbance as additional departures} \\
\text{annual revenues in millions} &= \text{widget sales} \times \text{widget price} / 1000000 \\
\text{average salary} &= 25000 \\
\text{budgeted size of sales force} &= \text{sales dept budget} \times 1000000 / \text{average salary} \\
\text{disturbance as additional departures} &= \text{IF}(\text{TIME} = \text{target year}) \text{ THEN number of departures ELSE 0} \\
\text{exit rate} &= 0.2 \\
\text{fraction to sales} &= 0.5 \\
\text{hiring fraction} &= 1 \\
\text{number of departures} &= \text{siza of sales force} \times 0.4 \\
\text{sales dept budget} &= \text{fraction to sales} \times \text{annual revenues in millions} \\
\text{target year} &= 15 \\
\text{widget price} &= 100 \\
\text{widget sales} &= \text{siza of sales force} \times \text{effectiveness in widgets per day} \times 365 \\
\text{effectiveness in widgets per day} &= \text{GRAPH} (\text{siza of sales force}) \\
&(0.00, 2.00), (200, 2.00), (400, 2.00), (600, 1.80), (800, 1.60), (1000, 0.8), (1200, 0.4)
\end{align*}
As we can see, an initial increase of the departures on year 15 triggers a substantial drop in size of sales force. New hires increases to accommodate the lower sales force until the size has reached higher levels.

We can verify this explanation by looking at the values in table form below.
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Among the three choices 50% is best. 55% fraction will kick the size of sales force into an oscillator as the size is growing too fast to accommodate a smooth transition. When the sales force grows too fast and the quality decreases suddenly, the oscillation occurs.
Exercise 6:

- Exponential decay (top left): exponential because it has arc from stock. Decay because it has outflow. Assumptions are that decay rate is > 0.
- Exponential Growth (bottom right): exponential because it has arc from stock. Growth because it has inflow. Assumptions that growth rate is > 0.
- S-Shaped Growth (bottom left): similar to lab problems. S shaped growth requires both growth and decay and that they will eventually be very similar as to facilitate equilibrium. Also a discrete specification of growth rate is necessary, otherwise you will not get an S shaped.
- Approach Equilibrium (top right): Although S shaped can also be correct. This model has constant inflow which is either bigger or smaller than the decay rate. If it’s bigger, then the stock will grow until the decay rate matches it and then it will be at equilibrium. If it’s lower, then stock will decrease exponentially.