1) \( y = \frac{x^3+2x^2-x-2}{x^3+x^2-4x-4} \)

Factor both the numerator and denominator and simplify \( y \) as much as possible.

**SOLUTION:** The numerator of \( \frac{x^3+2x^2-x-2}{x^3+x^2-4x-4} \) is extracted with numer. We then use factor to factor the result of executing the numer command.

\[
> \text{numer}\left((x^3+2x^2-x-2)/(x^3+x^2-4x-4)\right); \\
x^3 + 2x^2 - x - 2
\]

\[
> \text{factor}(x^3+2x^2-x-2); \\
(x-1)(x+2)(x+1)
\]

Similarly, we use denom to extract the denominator of the fraction. Again, factor is used to factor the denominator of the fraction.

Then, normal is used to find that

\[
\frac{x^2-1}{x^2-2x+1} = \frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1}.
\]

In this calculation, we have assumed that \( x \neq 1 \).

\[
> \text{convert}(1/((x-3)*(x-1)), \text{parfrac}, x); \\
1/2 \, (x - 3)^{-1} - 1/2 \, (x - 1)^{-1}
\]

\[
> \text{normal}\left((x^2-1)/(x^2-2*x+1)\right); \\
\frac{x+1}{x-1}
\]

2) Solve \( \sin^2 x - 2 \sin x - 3 = 0 \). Check by plugging in the solutions and plotting the function.

**SOLUTION:** When the command \( \text{solve(sin(x)^2-2*sin(x)-3=0)} \) is entered, Maple solves the equation for \( x \).

\[
> \text{solve(sin(x)^2-2*sin(x)-3=0)};
\]

\( \text{complex} (\text{arcsin}(3))-1/2\pi \)

However, when we set \_EnvAllSolutions := true, Maple attempts to solve the equation for all values of \( x \). In this case, the equation has infinitely many solutions of the form \( x = \frac{1}{2}(4k-1)\pi, k = 0, \pm 1, \pm 2, \ldots \); \( \sin x = 3 \) has no solutions.

\[
> \text{_EnvAllSolutions := true; } \\
> \text{solve(sin(x)^2-2*sin(x)-3=0);}
\]

\( \text{arcsin}(3) - 2 \text{ arcsin}(3) \_B1 + 2 \pi \_Z1 + \pi \_B1, -1/2\pi + 2\pi \_Z2 \)
First reset everything, and define our function:

\[
 f := x \rightarrow \sin(x)^2 - 2\sin(x) - 3
\]

Now ask Maple to solve it:
\[
 \text{zeros} := \text{solve}(f(x) = 0)
\]

\[
 \text{arcsin}(3), \ -\frac{1}{2}\pi
\]

These appear to be the correct roots, verify by plugging them in, individually, and then together as a sequence just for fun:

So we can see that indeed the function does equal 0 at \(-\frac{1}{2}\pi + 2\pi k \forall k \in \mathbb{Z}\).

\[
 \text{solve}(z^2 - 2z - 3)
\]

\[
 2, -1
\]

\[
 \sin(x) = -1 \quad \text{for} \quad x = \frac{\pi}{2} + \pi k, \ k \in \mathbb{Z}
\]

\[
 \sin(x) = 3 \quad \text{for} \quad \text{no solution} x = \arcsin(3)
\]
1) Approximate the solutions to the system of equations:
\[ x^2 + 4xy + y^2 = 4, \quad 5x^2 - 4xy + 2y^2 = 8. \]

First plot the functions using “contourplot”. Then try using “solve” and finally “fsolve”.

**SOLUTION:** We begin by using `contourplot` to graph each equation in Figure 2-36. From the resulting graph, we see that \( x^2 + 4xy + y^2 = 4 \) is a hyperbola, \( 5x^2 - 4xy + 2y^2 = 8 \) is an ellipse, and there are four solutions to the system of equations.

```plaintext
> with(plots):
> cp1:=contourplot(x^2+4*x*y+y^2=4,x=-4..4,y=-4..4,
> contours=[0],
> grid=[60,60],color=black):
> cp2:=contourplot(5*x^2-4*x*y+2*y^2=8,
> x=-4..4,y=-4..4,contours=[0],
> grid=[60,60],color=gray):
> display(cp1,cp2);
```

From the graph we see that possible solutions are \((0,2)\) and \((0,-2)\). In fact, substituting \( x = 0 \) and \( y = -2 \), and \( x = 0 \) and \( y = 2 \), into each equation verifies that these points are both exact solutions of the equation. The remaining two solutions are approximated with `fsolve`.

```plaintext
> fsolve(x^2+4*x*y+y^2=4,5*x^2-4*x*y+2*y^2=8,
> x=1.2,y=0..1):

\[ \{ x = 1.392621248, y = 0.3481553119 \} \]

> fsolve(x^2+4*x*y+y^2=4,5*x^2-4*x*y+2*y^2=8,
> x=-1.5..-1,y=-1.0):

\[ \{ y = -0.3481553119, x = -1.392621248 \} \]
```

![Figure 2-36](image)

*Figure 2-36*  Graphs of \( x^2 + 4xy + y^2 = 4 \) and \( 5x^2 - 4xy + 2y^2 = 8 \)
Start again by resetting our state and defining our functions:

```
restart
with(plots):

\[
f := (x, y) \rightarrow x^2 + 4 \cdot x \cdot y + y^2
\]

\[
(x, y) \rightarrow x^2 + 4 xy + y^2
\]

\[\tag{11}\]

\[
g := (x, y) \rightarrow 5 \cdot x^2 - 4 \cdot x \cdot y + 2 y^2
\]

\[
(x, y) \rightarrow 5 x^2 - 4 xy + 2 y^2
\]

\[\tag{12}\]

Now draw the contour plot:

\[
plotf := \text{contourplot}(f, -3..3, -3..3, \text{contours} = [4, \text{color} = \text{blue}]):
\]

\[
plotg := \text{contourplot}(g, -3..3, -3..3, \text{contours} = [8, \text{color} = \text{red}]):
\]

3
display(plotf, plotg)

It appears that the system is satisfied at $\approx \pm (0, 2)$ and $\pm (1.4, 0.35)$. We can plug these in to see:

\[
sols := \begin{bmatrix} [0, 2], & - [0, 2], & [1.4, 0.35], & - [1.4, 0.35] \end{bmatrix}:
\]
\[
map(f@op, sols), \quad map(g@op, sols)
\]
\[
\begin{bmatrix} 4, 4, 4.0425, 4.0425 \end{bmatrix}, \begin{bmatrix} 8, 8, 8.0850, 8.0850 \end{bmatrix}
\]

(13)

It looks as though the first two solutions were exact, and $\pm (1.4, 0.35)$ was close but a little bit off. So now try to get a numerical solution from maple, in this case we need to specify a starting position for the numerical solver so that it does not always return the same solution.

\[
sol_1 := fsolve( \{f(x, y) = 4, \ g(x, y) = 8\}, \{x = 0, y = 2\})
\]
\[
\{x = 0., y = 2.\}
\]

(14)

\[
sol_2 := fsolve( \{f(x, y) = 4, \ g(x, y) = 8\}, \{x = 0, y = -2\})
\]
\[
\{x = 0., y = -2.\}
\]

(15)
\[ \text{solsf}_3 := \text{fsolve}\left\{ f(x,y) = 4, \quad g(x,y) = 8 \right\}, \ (x = 1.4, \ y = 0.35) \right\} \]
\[ \{x = 1.392621248, \ y = 0.3481553119\} \quad (16) \]
\[ \text{solsf}_4 := \text{fsolve}\left\{ f(x,y) = 4, \quad g(x,y) = 8 \right\}, \ (x = -1.4, \ y = -0.35) \right\} \]
\[ \{x = -1.392621248, \ y = -0.3481553119\} \quad (17) \]
\[ \text{map}(\text{subs}, \text{convert}(\text{solsf}, \text{list}), \ f(x,y)), \ \text{map}(\text{subs}, \text{convert}(\text{solsf}, \text{list}), \ g(x,y)) \]
\[ [4., 4., 4.000000001, 4.000000001, 8., 8., 8.000000002, 8.000000002] \quad (19) \]

Apparently the results are somewhat better, though still not exact.
So finally try to find an exact solution:
\[ \text{solsf} := \text{solve}\left\{ f(x,y) = 4, \quad g(x,y) = 8 \right\}, \ (x,y) \right\} \]
\[ x = 8 \text{ RootOf}[33 \cdot Z^2 - 1, \ label = _L3], \ y = 2 \text{ RootOf}[33 \cdot Z^2 - 1, \ label = _L3] \}, \ (x = 0, \ y = 2) \,
\{x = 0, \ y = -2\} \quad (20) \]

So the exact solutions are \( \pm (0, 2) \) as we knew, and \( r \cdot (8, 2) \) where \( r \) is the roots of \( 33 \cdot z^2 - 1 \), which we can approximate:
\[ r := \text{evalf}(\text{RootOf}(33 \cdot Z^2 - 1)) \]
\[ 0.1740776560 \quad (21) \]
\[ r \cdot (8, 2) \]
\[ 1.392621248, 0.3481553120 \quad (22) \]
\[ -r \cdot (8, 2) \]
\[ -1.392621248, -0.3481553120 \quad (23) \]
As expected.