1. Recall the recursive function $C(n, k)$ in the class BinomialCoefficients discussed in lecture and posted on the webpage. Write a box trace of the function call $C(5, 3)$. Use this trace to find the value of $C(5, 3)$. Notice that in the full recursion tree for $C(5, 3)$, the value $C(3, 2)$ is evaluated 2 times, and $C(2, 1)$ is evaluated 3 times. Suggest a modification to the function that would allow it to avoid computing the same values multiple times. (Don’t write the code, just explain it in words.)

**Solution to second question:**

**Suggested modification:** when $C(n, k)$ is computed for the first time (for a particular $n$ and $k$), save the value in a static 2-dimensional array for later re-use. If the value $C(n, k)$ is needed at some later time, look it up in the array instead of computing it again.

Even though the question specifically says to not write the code, it's a nice exercise to do it. The simplest way to do this is to pass an array variable to the recursive function, as follows.

```java
static int C(int n, int k, int[][] BinCoef){
    if( BinCoef[n][k]!=0 ){
        return BinCoef[n][k];
    }else if( k==0 || k==n ){
        return 1;
    }else{
        BinCoef[n][k] = C(n-1,k-1, BinCoef)+C(n-1,k, BinCoef);
        return BinCoef[n][k];
    }
}
```

The calling function of this recursive function must allocate the array $BinCoef[][]$ and initialize it to all zeros. As an exercise, envelope this function in a class, and write a `main()` function to test it.

2. Write a recursive function called `sum(n)` that computes the sum of the integers from 1 to $n$. Hint: recall the recursive function `fact(n)` in the class Factorial discussed in lecture and posted on the webpage. Modify your answer so as to recursively compute the sum of the integers from $n$ to $m$, where $n \leq m$. (If $n > m$, return 0.)

**Solution to second question:**

```java
static int sum(int n, int m){
    if( n<=m ){
        return sum(n, m-1) + m;
    }else{
        return 0;
    }
}
```
3. Write a recursive function called `sumArray()` that determines the sum of the integers in an array \( A[0...n-1] \). Do this in 3 ways.
   a. Recur on \( A[0...n-2] \), add the result to \( A[n-1] \), then return the sum.
   b. Recur on \( A[1...n-1] \), add the result to \( A[0] \), then return the sum.
   c. Split \( A[0...n-1] \) into two subarrays of length (approximately) \( n/2 \), recur on the two subarrays, add the results and return the sum. Hint: think about `MergeSort()`.

**Solution to part c:**
```
static int sumArray(int[] A, p, r){
    if( p<r ){
        int q = (p+r)/2;
        int a = sumArray(A, p, q);
        int b = sumArray(A, q+1, r);
        return a+b;
    }else if( p==r ){
        return A[p];
    }else{
        return 0;
    }
}
```

4. Write a modification of the recursive function `BinarySearch()` that prints out the sequence of array elements that are compared to the target.

**Solution:**
```
static int BinarySearch(int[] X, int p, int r, int target){
    if( p<=r ){
        int q = (p+r)/2;
        System.out.print(X[q]+" ");
        if( target==X[q] ){
            return q;
        }else if( target<X[q] ){
            return BinarySearch(X, p, q-1, target);
        }else{ // target>X[q]
            return BinarySearch(X, q+1, r, target);
        }
    }else{
        return -1;
    }
}
```
7. Use what you learned in problem 6 to create a recursive function called `integerToString()` that returns a String representation of an integer \( n \) expressed in base \( b \). For instance the function call `integerToString(100, 8)` would return the String “144”, which is what was printed in problem 6.

**Solution:**
The following full program defines the required function, along with a helper function that produces digits in various bases greater than 10, and tests the functions on various bases.

```java
class Problem7{

    static String integerToString(int n, int b){
        String s="";
        if( n>0 ){
            if( n>=b ){
                s = integerToString(n/b, b);
            }
            return s + digit(n%b, b); // String.valueOf(n%b);
        }else{
            return s;
        }
    }

    static String digit(int d, int b){
        if( d<0 || d>=b ){
            System.err.println(d+" is not a digit in base "+b);
            System.exit(1);
        }
        if( d<10 ){
            return String.valueOf(d);
        }else{
            return String.valueOf((char)(d+55));
        }
    }

    public static void main(String[] args){
        for(int b=2; b<=100; b++){
            System.out.println("base = "+b+"\t"+integerToString(43981,b));
        }
    }
}
```
8. Recall the IntegerList ADT discussed in class whose states were the finite integer sequences, and whose operations were `isEmpty()`, `size()`, `get()`, `add()`, `remove()`, and `removeAll()`. Write the methods described below using only these six ADT operations. In other words you are writing methods belonging to a client of `IntegerList`.

a. Write a static void method called `swap(IntegerList L, int i, int j)` that will interchange the items currently at positions `i` and `j` of the List.

b. Write a static int method called `search(IntegerList L, int x)` that will perform a linear search of `L` for the target `x`. `search()` will return the List index where `x` was found, or it will return 0 if no such index exists. (Recall List indices range from 1 to `size()`.)

c. Write a static void method called `reverse(IntegerList L)` that reverses the order of the items in `L`.

Solution to part a:
```java
static void swap(IntegerList L, int i, int j){
    int a = L.get(i);
    int b = L.get(j);
    L.remove(i);
    L.add(i, b);
    L.remove(j);
    L.add(j, a);
}
```

Solution to c:
```java
static reverse(IntegerList L){
    int i=1, j=L.size();
    while( i<j ){
        swap(L, i, j);
        i++;
        j--;
    }
}
```