Homework #7: Lead, Lag, and Notch Filter design.

Given the system: \( G(s) = \frac{1}{s^2 + 1} \). Note, you may use MATLAB to check your work, but I want you to do most of this “by hand” in order to develop your intuition.

1. Design a lead controller with its zero at the origin, \( K(s) = K \frac{s}{s + p} \) (unity feedback) such that the dominant second order roots are at \( s = -2 \pm 2j \). Use “root locus techniques” to do this design. What is the resulting \( K(s) \)?

2. Sketch the locus of roots against the loop gain for the design generated in (1) above. Show asymptotes, center of asymptotes, departure angles, etc. where appropriate.

3. Repeat (1) and (2) above, but this time placing the pole of the lead controller at negative \( \infty \), that is \( K(s) = K(s + z) \)

4. Compare the pros and cons of these two designs. In particular, sketch the response of the system to a step input. Also, remember that we’ve told you that compensators with pure zeros can be problematic due to the fact that they differentiate noise.

Assume that the system is really: \( G(s) = \frac{36}{(s + 1)(s^2 + 0.5s + 36)} \).

5. Use the controller you designed in part (3) and redraw the locus. Where are the closed-loop roots now (assuming no change to the compensator)? At what value of the loop gain with the system go unstable?

6. Add a notch filter to your compensator, \( K(s) \). Place the zeros of the notch such that the departure angle from the high frequency poles is 180 degrees. What is the new \( K(s) \)? Sketch the new root locus.