Dithering

The Problem

• An image has $N$ intensity levels, and the screen or hardcopy device has only $n<N$ levels. How can we reproduce the image in a satisfactory way?
• Quantization won’t work well when $n=2$
• Solution: halftoning or dithering
Halftoning (Example)

Halftoning

- Array of circles with diameter proportional to blackness (1-Intensity).
- Human visual system performs spatial integration (interpolation)
  - If we view a very small area from a sufficiently large distance, our eyes average fine details and record only the overall intensity area.
- Newspapers: 60-80 variable-sized and variable-shaped areas per inch.
- Magazine and books: 110-200 per inch.
Dithering

- Assume the original image has only \( k^2 + 1 \) intensity levels.
- Each pixel of the original image is mapped into a \( k \times k \) block of the **dithered image**.
- Example: \( k=2 \)

\[
\begin{array}{c|c|c|c|c|c}
0 & 1 & 2 & 3 & 4 \\
\hline
\[
\begin{array}{c|c|c|c|c|c}
\end{array}
\end{array}
\]

- **Problem:** we reduce image resolution by \( 1/k \)
  (because each pixel is “zoomed” into a \( k \times k \) block)
  – It’s no problem if the display array is larger than the image array
Dither Matrix

- Dithering is controlled by the dither matrix \( D \).
  For example, with \( k=3 \):
  \[
  D = \begin{pmatrix}
  6 & 8 & 4 \\
  1 & 0 & 3 \\
  5 & 2 & 7
  \end{pmatrix}
  \]
  To display an intensity \( I \) (with \( 0 \leq I \leq 9 \)), just “turn on” all pixels in the 3x3 block whose values are less than \( I \).
- Must be carefully designed to avoid artifacts
  - What happens with \( D = \begin{pmatrix}
  6 & 8 & 4 \\
  1 & 0 & 2 \\
  5 & 3 & 7
  \end{pmatrix} \) and constant \( I=3 \)?

Multi-level Dithering

- What if we have a display with \( n>2 \) levels?
- We can still use dithering to increase the number of “effective” intensity levels: the intensity \( I \) is approximated by the sum of the values in a \( k \times k \) neighborhood.
- We can achieve \((n-1) \cdot k \cdot k + 1\) “effective” intensity levels.
Keeping the Same Resolution

• Each image point \((x,y)\) should control only the \((x,y)\) display pixel

• A possible technique:
  – Compute \(i=(x \mod k) + 1, j=(y \mod k) + 1\)
  – Turn on the pixel at \((x,y)\) only if \(I(x,y)>D(i,j)\), where \(D(i,j)\) is the entry of \(D\) in the \(i\)-th row and \(j\)-th column

Error Diffusion

• For each pixel \((x,y)\) of the original image \(I\), scanned in raster order (left to right, top to bottom):
  – Quantize \(I(x,y)\) to \(Q(x,y)\)
  – \(e(x,y)=I(x,y)-Q(x,y)\) is the error at \((x,y)\)
  – “Diffuse” the error:
    • \(I(x+1,y) += 7e/16\) [right]
    • \(I(x-1,y+1) += 3e/16\) [below left]
    • \(I(x,y+1) += 5e/16\) [below]
    • \(I(x+1,y+1) += e/16\) [below right]
  – Move to the next pixel
Error Diffusion (Example)

Error Diffusion for Color Images

• Error diffusion can be used also for color images:
  – Need to define the **quantization** operator
    • E.g., quantize to the color vector in the colormap with the minimum Euclidean distance
  – Need to define the **diffusion** strategy
    • E.g., add the same proportion of the error to the three channels separately
Example

- Original image
- Quantization with 5 bits (adaptive colormap)
- Quantization with 5 bits (adaptive colormap + error diffusion)

References