CMPE-242
Applied Feedback Control

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Questions?

\[ \text{Step (GAL):} \]

Eigenvalue: Square Matrix \( A \)

\[
A x_0 = \lambda x_0 \quad x_0 = 0
\]

\[
(\lambda - \lambda I) x_0 = 0
\]

\( \lambda \)'s "eigenvalue"

\( x_0 \) "eigenvector"
SVD - Singular Value Decomposition

\[ A \rightarrow U \Sigma V^T \]

\[ \sigma_1 \sigma_2 \ldots \sigma_n \]

\[ \mathbb{R}^n \rightarrow A \rightarrow \mathbb{R}^m \]

\[ \mathbb{R}^n \rightarrow V \rightarrow \Sigma \rightarrow U^T \rightarrow \mathbb{R}^m \]
STATE SPACE CONTROL

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

\[
u = -Kx\]

"Full state feedback"

Choose \( K \) to put \( \text{eig}(A-BK) \) where I desire them

Check \( \text{rank}(C) \), \( \text{rank}(C) \)

\[
C = [B \; AB \; A^2B \; \ldots \; A^{n-1}B]
\]
\[ k(T) = \frac{\sigma_{\text{max}}(T)}{\sigma_{\text{min}}(T)} \]  

"condition number"  

\[ \text{cond} \left( \frac{1}{k} \right) \sim 1 \rightarrow 0. \]
**Modal Coordinates**

\[ \dot{x} = Ax + Bu \rightarrow x = T_3; \quad z = T'x \]

\[ T^{-1}AT = \begin{bmatrix} \lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \lambda_n \end{bmatrix} \]

4 Block Jordan Form

\[
\begin{bmatrix}
-2\nu_i^2 & 2
\
-\omega_n^2 & -1
\end{bmatrix}
\]

2 Modes drag and oscillate

[Signature: Gabriel Hugh Elkaim]
\[
\tilde{\mathbf{x}}_t = \mathbf{A} \mathbf{x}_t + \mathbf{B} \mathbf{u}_t - \mathbf{c}
\]

\[
\begin{bmatrix}
1 & 0 \\
0 & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]
\[ \dot{x} = Ax + Bu \]

\[ x(t) = e^{At} \]

\[ e^{At} = I + At + \frac{A^2 t^2}{2} + \frac{A^3 t^3}{3!} + \ldots \exp \mu \]

\[ e = \begin{bmatrix} \Sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Sigma_n \end{bmatrix} \]
\( C = [B \ xB \ x^2B \ \cdots \ x^{n-1}B] \)

All rank = rank \( (C) = n \) “controllable”

\( K(C) = \frac{\sigma_{\text{max}}(C)}{\sigma_{\text{min}}(C)} \to \infty \) “approaching singular”
\[ x = x x + Bu \quad x = T_3 \]

\[ \dot{x} = T'xT_3 + T'Bu \]

\[ \dot{C}_m = \left[ \begin{array}{c}
-\dot{\gamma}B \\
-\dot{\gamma}T'\beta \\
-\dot{\gamma}T'\alpha T'\beta \\
\end{array} \right] \]

\[ \frac{1}{T'} = \text{Cold} \]
\[
\dot{x} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\
y = \begin{bmatrix} 0 & 1 \end{bmatrix} x + [0] u
\]

I: \( B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)

C: \( \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} \)

\[ \text{rank}(C) = 2 \]

\[ \text{cod}(C) = 2 \]

\[ C \begin{bmatrix} 55 - \lambda \\ 1 \end{bmatrix} B \rightarrow \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda + 3 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\lambda + 3}{(\lambda + 2)(\lambda + 1)} \]

II: \( B = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \)

C: \( \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \)

\[ \text{rank}(C) = 1 \]

\[ \text{cod}(C) = 0 \]
\[ W = \begin{bmatrix} -0.999 & 0.997 \\ 1 & 1 \end{bmatrix} \]

\[ C = \begin{bmatrix} 1 & -0.999 \\ 0.997 & 1 \end{bmatrix} \]

\[ \text{rank}(C) = 2 \]

\[ \text{cond}(C) = 4000 \]

\[ C(sI-A)^{-1} = \frac{s+2.001}{(s+2)(s+1)} \]
\[ \dot{x} = Ax + Bu \]
\[ u = -k(x - x_e) \]
\[ y = Cx + Du \]

If I have \( x \) and \( C \) is full rank I have very good control.
\[
x(t) = Ax + Bu \\
y(t) = Cx + Du
\]

\[
u(t) = -K(x - x_d)
\]

\[
\begin{align*}
\hat{x} &= A\hat{x} + Bu \\
\hat{y} &= C\hat{x}
\end{align*}
\]

\[
u(t) = -K(\hat{x} - x_d)
\]
\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx \\
u &= -K\hat{x}
\end{align*}
\]

\[
\begin{align*}
\hat{x} &= \hat{x} + Bu + L(y - \hat{y}) \\
\hat{y} &= C\hat{x}
\end{align*}
\]
\[ \dot{x} = A_0 x + B_0 u \quad u = -k \hat{x} \]

\[ y = C_0 x \quad \text{Control gain} \]

\[ \hat{x} = A_0 \hat{x} + B_0 u + L(y - \hat{y}) \quad \text{Estimator gain} \]

\[ \check{x} = (A - BK) \hat{x} + L(y - \hat{y}) \]

\[ \hat{x} = \check{x} - \hat{x} = A_0 x + B_0 u - A \hat{x} - B \hat{x} - L(y - \hat{y}) \]

\[ = A \check{x} - LC_0 x + LCC_0 \hat{x} \]

\[ = A \check{x} - LC_0 (x - \hat{x}) = (A - LC_0) \check{x} \]
\[(A-BK) \text{ control gain}\]
\[
\begin{bmatrix}
1 & 1
\end{bmatrix}
\]

\[K = \text{place}(A, B, P_{d,s})\]

\[(A-(C) \text{ estimator gain}\]
\[
\begin{bmatrix}
1 & 1
\end{bmatrix}
\]

\[\Lambda^T - C^T L^T \quad \text{"dual" } A-BK\]

\text{estima}
For control

\[ C = \begin{bmatrix} B & AB & A^2B & \ldots & A^{n-1}B \end{bmatrix} \quad \text{full rank calculable} \quad \text{cond}(C) \]

\[ C_{\text{obs}} = \begin{bmatrix} c^T & A^1c^T & (A^2)^1c^T & \ldots & (A^{n-1})^{n-1}c^T \end{bmatrix} \]

\[ \Phi = \begin{bmatrix} c & Ac & A^2c & \ldots & A^{n-1}c \end{bmatrix} \]

observability matrix \( \text{obsv}(\alpha, c) \)

full rank - observable

\[ \text{cond}(\Phi) \]
The diagram shows a control system with a feedback loop.

- The input is denoted by $f$.
- The output is denoted by $u$.
- The controller is denoted by $K(s)$.
- The system has a $n$th order controller.
- The $2n$th order open loop is also shown.

The equation $u = -kx$ is also shown, indicating a simple proportional control action.
\[ \dot{\hat{x}} = A\hat{x} + Bu + C(y - \hat{y}) \]
\[ u = -K\hat{x} \]
\[ \hat{y} = C\hat{x} \]

\[ \frac{C}{\hat{y}(s)} = \frac{K(s)}{\hat{y}(s)} \]

\[ \dot{\hat{x}} = (A - LC - BK)\hat{x} + C(y - \hat{y}) \]
\[ u = -K\hat{x} \]

\[ \frac{C(sI - A + BK + LC)}{sI - A + BK + LC} \]

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\[ u = -K \hat{x} \quad \text{plane poles in } P_{\text{as}}, \quad \text{(choose } k) \]

\[ \bar{L} = \text{plane span in } P_{\text{as}} \quad \bar{x} = \hat{x} - \hat{x} \]

\[ \hat{x} = (A - LC) \hat{x} \]

\[ \dot{x} = Ax - BK \hat{x} = Ax - BK(x - \hat{x}) \]

\[ = (A - BK)x + BK \hat{x} \]

\[ \begin{bmatrix} \dot{x} \\
\dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\
0 & A - LC \end{bmatrix} \begin{bmatrix} x \\
\hat{x} \end{bmatrix} \]

"Separation principle"

\[ \text{det} \left[ \frac{\partial}{\partial t} \right] = \text{det}(A - BK) + \text{det}(A - LC) \]
\[
\begin{bmatrix}
\hat{\theta} \\
\hat{\dot{\theta}}
\end{bmatrix}
\quad \quad u = -k_x
\]

Un estimator del \( \dot{\theta} \)
\[ \dot{x} = (k - uc - 8k)\hat{x} + cy \]

\[ \hat{x} = (k - uc)\hat{x} + Bu + cy \]
(1) Measure $y$

(2) $y - L \rightarrow \text{Form } \hat{x}$

(3) $\dot{x} = [2] + [...] \hat{x} + [r_{\text{cmd}}]$

(4) $\dot{x}^+ = \dot{x}^- + \dot{x}^* \Delta T$

(5) $u_{\text{cmd}} = K \ddot{x}^+$