1. **Differential Equations:** Consider the system defined by:

\[ m\ddot{x} + b\dot{x} = F(t) \] where \( x(0) = 0 \) and \( \dot{x}(0) = 0 \).

a. Find the step response of this system. That is, what is \( x(t) \) if \( F(t) = 1(t) \). Do this in a way that does NOT use Laplace Transforms. For example, you can solve the equation using the techniques you learning in your first DiffEq class (i.e.: find the forced and natural parts of the response, etc.). So that you might check this, the answer is \( x(t) = \frac{1}{b} (t - \frac{m}{b} \left( 1 - e^{-\frac{b}{m}t} \right)) \).

b. The impulse response of the system, \( h(t) \), is the derivative of the step response. What is \( h(t) \)?

c. Compute the step response using the convolution integral \( x(t) = \int_{0}^{t} u(\tau) h(t - \tau) d\tau \), where \( u(\tau) = 1 \).

d. What is the transfer function \( \frac{X(s)}{F(s)} \) of this system? What is the Laplace Transform of a step input, \( U(s) \)?

e. Find the step response, \( x(t) \), by taking the inverse Laplace Transform of \( X(s) \). Use the table of Laplace Transforms.

f. Repeat part (e), but first do a partial fraction expansion of \( X(s) \). Note that there are repeated roots (see Appendix A of FPE).

2. **Block diagram reduction:** Write down the transfer function \( \frac{Y(s)}{U(s)} \) of the block diagram below:
3. **Laplace transform**: find the time function for each of the following using the Inverse Laplace Transforms and partial fraction expansion (look at Appendix A for distinct complex roots):

   a. \( F(s) = \frac{2}{s(s+2)} \)
   
   b. \( F(s) = \frac{3s+2}{s^2+4s+20} \)
   
   c. \( F(s) = \frac{2(s+2)}{(s+1)(s^2+4)} \)

4. **Transfer Functions**: Given the following mass-spring system, derive the transfer function from the position of the leftmost of the masses to the forcing function \( X_1(s)/F(s) \):

   ![Mass-Spring System Diagram](image)

5. **Transfer Functions**: Given the following simple electrical system, derive the transfer function from the input voltage to the output voltage \([V_2(s)/V_1(s)]\):

   ![Electrical System Diagram](image)

6. **Dynamic Response**: Given the following third order system:

   \[
   H(s) = \frac{\alpha \omega_n^2}{(s + \alpha)(s^2 + 2\zeta \omega_n s + \omega_n^2)}
   \]

   The response to a unit step \((U(t) = 1(t))\) is:

   \[
   y(t) = 1 + Ae^{-\alpha t} + Be^{-\alpha t} \sin(\omega_d t - \varphi)
   \]

   where:

   \[
   A = \frac{-\omega_n^2}{(\omega_n^2 - 2\zeta \omega_n \alpha + \alpha^2)}
   \]
\[
B = \frac{\alpha}{\sqrt{(\omega_n^2 - 2\zeta \omega_n \alpha + \alpha^2)(1 - \zeta^2)}}
\]

\[
\varphi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta} + \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta \omega_n}
\]

a. Which term dominates as \( \alpha \) gets large?

b. Which term dominates as \( \alpha \) gets small?

c. Approximate \( A \) and \( B \) for small values of \( \alpha \).

d. Assume that \( \omega_n = 1 \) and \( \zeta = 0.707 \), plot the step response for several values of \( \alpha \). Use MATLAB's `step` command (could you use `impulse`? How?) Comment on where the extra pole becomes unimportant.

e. **Extra credit:** show that this is, indeed, the response to the step input (lots of hairy complex math and trig transformations).