1. **Digital and Continuous Equivalents (Revisited):** Consider the following simple oscillatory system, with two poles on the \( j\omega \) axis:

\[
G(s) = \frac{3}{s^2 + 3}
\]

a. Find the ZOH equivalent of \( G(s) \), \( G(z) \) given a sampling rate of 4Hz (\( \Delta T = 0.25 \) seconds).

b. Analyze the designs you created in Problem 5-1(a-d), mapping the z-plane roots back to the s-plane using the exact equivalent: \( s = \frac{1}{\Delta T} \ln z \). How to the characteristics compare to what you were trying to achieve? Comment on any differences you notice.

c. Design a new controller, \( K_3(z) \), this time designing directly in the z-domain. Use the desired z-plane pole locations to be \( z_{des} = e^{s_{des}\Delta T} \), then use root locus techniques to find the compensator that will put your closed loop poles there.

d. Using MATLAB, plot the step responses of all of the systems on the same plot (the ideal continuous one, and all of the digital variants). Make sure to align the time axis correctly.

e. Using MATLAB, plot the impulse responses of all of the compensated systems on the same plot (the ideal continuous one, and all of the digital variants). Make sure to align the time axis correctly.

f. Comment on how each of the designs works.
2. *The Zero Order Hold:* Consider the following system with the simplest of compensators 
\[ K(z) = 1, \] 
with a sample time of \( \Delta T = 0.25 \) sec, and remember that the Laplace Transform of the ZOH is 
\[ \mathcal{L}\{\text{ZOH}\} = \frac{1 - e^{-\Delta Ts}}{\Delta Ts}. \]

- a. Compute the frequency response for \( \omega=0 \) to the nyquist sampling frequency (2Hz). Use MATLAB to generate this with a `for` loop.
- b. Use SIMULINK to simulate the experience of probing the system with a function generator and an o-scope (that is, be able to tune any sine wave in, and read the output in terms of phase and magnitude).
- c. Show that the output of the magnitude is a sinc function: 
\[ |G(e^{j\omega \Delta T})| = \frac{\sin(\omega \Delta T/2)}{\omega \Delta T/2}. \]
- d. Show that the phase lag is linear in phase: 
\[ \arg G(e^{j\omega \Delta T}) = -\frac{\omega \Delta T}{2}. \]