Basic Notation

Basic set notation

\{a_1, \ldots, a_r\} \quad \text{the set with elements } a_1, \ldots, a_r.

\(a \in S\) \quad \text{a is in the set } S.

\(S = T\) \quad \text{the sets } S \text{ and } T \text{ are equal, i.e., every element of } S \text{ is in } T \text{ and every element of } T \text{ is in } S.

\(S \subseteq T\) \quad \text{the set } S \text{ is a subset of the set } T, \text{ i.e., every element of } S \text{ is also an element of } T.

\exists a \in S \ P(a) \quad \text{there exists an } a \text{ in } S \text{ for which the property } P \text{ holds.}

\forall x \in S \ P(a) \quad \text{property } P \text{ holds for every element in } S.

\{ a \in S \mid P(a) \} \quad \text{the set of all } a \text{ in } S \text{ for which } P \text{ holds (the set } S \text{ is sometimes omitted if it can be determined from context.)}

\(A \cup B\) \quad \text{union of sets, } A \cup B = \{ x \mid x \in A \text{ or } x \in B \}.

\(A \cap B\) \quad \text{intersection of sets, } A \cap B = \{ x \mid x \in A \text{ and } x \in B \}\.

\(A \times B\) \quad \text{Cartesian product of two sets, } A \times B = \{ (a, b) \mid a \in A, \ b \in B \}.

Some specific sets

\(\mathbb{R}\) \quad \text{the set of real numbers.}

\(\mathbb{R}^n\) \quad \text{the set of real } n\text{-vectors} (n \times 1 \text{ matrices}).

\(\mathbb{R}^{1 \times n}\) \quad \text{the set of real } n\text{-row-vectors} (1 \times n \text{ matrices}).

\(\mathbb{R}^{m \times n}\) \quad \text{the set of real } m \times n \text{ matrices.}

\(j\) \quad \text{can mean } \sqrt{-1}, \text{ in the company of electrical engineers.}

\(i\) \quad \text{can mean } \sqrt{-1}, \text{ for normal people; } i \text{ is the polite term in mixed company (i.e., when non-electrical engineers are present.)}

\(\mathbb{C}, \mathbb{C}^n, \mathbb{C}^{m \times n}\) \quad \text{the set of complex numbers, complex } n\text{-vectors, complex } m \times n \text{ matrices.}

\(\mathbb{Z}\) \quad \text{the set of integers: } \mathbb{Z} = \{ \ldots, -1, 0, 1, \ldots \}.

\(\mathbb{R}_+\) \quad \text{the nonnegative real numbers, i.e., } \mathbb{R}_+ = \{ x \in \mathbb{R} \mid x \geq 0 \}.

\([a, b], (a, b], [a, b), (a, b)\) \quad \text{the real intervals } \{ x \mid a \leq x \leq b \}, \{ x \mid a < x \leq b \}, \{ x \mid a \leq x < b \}, \text{ and } \{ x \mid a < x < b \}, \text{ respectively.}
Vectors and matrices

We use square brackets $[\text{ and }]$ to construct matrices and vectors, with white space delineating the entries in a row, and a new line indicating a new row. For example $[1 \ 2]$ is a row vector in $\mathbb{R}^{2 \times 1}$, and $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is matrix in $\mathbb{R}^{2 \times 3}$. $[1 \ 2]^T$ denotes a column vector, i.e., an element of $\mathbb{R}^{2 \times 1}$, which we abbreviate as $\mathbb{R}^2$.

We use curved brackets $(\text{ and })$ surrounding lists of entries, delineated by commas, as an alternative method to construct (column) vectors. Thus, we have three ways to denote a column vector:

$$(1, 2) = [1 \ 2]^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$  

Note that in our notation scheme (which is fairly standard), $[1, 2, 3]$ and $(1 \ 2 \ 3)$ aren’t used.

Functions

The notation $f : A \rightarrow B$ means that $f$ is a function on the set $A$ into the set $B$. The notation $b = f(a)$ means $b$ is the value of the function $f$ at the point $a$, where $a \in A$ and $b \in B$. The set $A$ is called the domain of the function $f$; it can thought of as the set of legal parameter values that can be passed to the function $f$. The set $B$ is called the codomain (or sometimes range) of the function $f$; it can thought of as a set that contains all possible returned values of the function $f$.

There are several ways to think of a function. The formal definition is that $f$ is a subset of $A \times B$, with the property that for every $a \in A$, there is exactly one $b \in B$ such that $(a, b) \in f$. We denote this as $b = f(a)$.

Perhaps a better way to think of a function is as a black box or (software) function or subroutine. The domain is the set of all legal values (or data types or structures) that can be passed to $f$. The codomain of $f$ gives the data type or data structure of the values returned by $f$.

Thus $f(a)$ is meaningless if $a \not\in A$. If $a \in A$, then $b = f(a)$ is an element of $B$. Also note that the function is denoted $f$; it is wrong to say ‘the function $f(a)$’ (since $f(a)$ is an element of $B$, not a function). Having said that, we do sometimes use sloppy notation such as ‘the function $f(t) = t^3$’. To say this more clearly you could say ‘the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(t) = t^3$ for $t \in \mathbb{R}$’.
Examples

• $-0.1 \in \mathbb{R}$, $\sqrt{2} \in \mathbb{R}_+$, $1 - 2j \in \mathbb{C}$ (with $j = \sqrt{-1}$).

• The matrix

$$A = \begin{bmatrix} 0.3 & 6.1 & -0.12 \\ 7.2 & 0 & 0.01 \end{bmatrix}$$

is an element in $\mathbb{R}^{2 \times 3}$. We can define a function $f : \mathbb{R}^3 \to \mathbb{R}^2$ as $f(x) = Ax$ for any $x \in \mathbb{R}^3$. If $x \in \mathbb{R}^3$, then $f(x)$ is a particular vector in $\mathbb{R}^2$. We can say ‘the function $f$ is linear’. To say ‘the function $f(x)$ is linear’ is technically wrong since $f(x)$ is a vector, not a function. Similarly we can’t say ‘$A$ is linear’; it is just a matrix.

• We can define a function $f : \{ a \in \mathbb{R} \mid a \neq 0 \} \times \mathbb{R}^n \to \mathbb{R}^n$ by $f(a, x) = (1/a)x$, for any $a \in \mathbb{R}$, $a \neq 0$, and any $x \in \mathbb{R}^n$. The function $f$ could be informally described as division of a vector by a nonzero scalar.

• Consider the set $A = \{ 0, -1, 3.2 \}$. The elements of $A$ are 0, $-1$ and 3.2. Therefore, for example, $-1 \in A$ and $\{ 0, 3.2 \} \subseteq A$. Also, we can say that $\forall x \in A\, -1 \leq x \leq 4$ or $\exists x \in A\, x > 3$.

• Suppose $A = \{ 1, -1 \}$. Another representation for $A$ is $A = \{ x \in \mathbb{R} \mid x^2 = 1 \}$.

• Suppose $A = \{ 1, -2, 0 \}$ and $B = \{ 3, -2 \}$. Then

$$A \cup B = \{ 1, -2, 0, 3 \}, \quad A \cap B = \{ -2 \}.$$ 

• Suppose $A = \{ 1, -2, 0 \}$ and $B = \{ 1, 3 \}$. Then

$$A \times B = \{ (1,1), (1,3), (-2,1), (-2,3), (0,1), (0,3) \}.$$ 

• $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2 - x$ defines a function from $\mathbb{R}$ to $\mathbb{R}$ while $u : \mathbb{R}_+ \to \mathbb{R}^2$ with

$$u(t) = \begin{bmatrix} t \cos t \\ 2e^{-t} \end{bmatrix}.$$ 

defines a function from $\mathbb{R}_+$ to $\mathbb{R}^2$. 

---

courtesy of Stephen Boyd @ Stanford University