Quite a solution

\[ \sum_{k=0}^{n} \frac{a^k b^{n-k}}{k! (n-k)!} = (a+b)^n = 10^6 \]

\[ q' = \frac{1}{5} q \Rightarrow q(t) = Ce^{\frac{t}{5}} \quad C = 1 \quad \frac{t}{5} > \ln 4 \Rightarrow t > 5 \ln 4 \]

Event

Random variable: Assigns value to outcomes

Random experiment: Outcome not known in advance

but set of all outcomes is known.

Possible

10 devices

if all polled = 1 cycle

if one needs service, service happens

Outcome of polls in one cycle \([1, 10]\\)

Sample set of all possible outcomes = \(\Omega\\)

Each outcome is an event or sample point

Examples:

\(\Omega = \{1, 2, 3, 4, 5\}\\)

\(\Omega = \{x : Q(x) \}\\)

such that \(Q(x)\\) holds
1. Coin tossing \( \mathcal{S} = \{1, 2, 3, \ldots, 5, 6\} \)

2. 2 fair dice

3. \( \mathcal{S} = \{(1, 1), (1, 2), \ldots, (6, 6)\} \)

4. \( \mathcal{S} = \{0, 1\} \) binary outcome

Design sample space so that you have more information.

3. \( \exists \) devices on com line \( \circ \circ \circ \circ \circ \)

4. Measuring response time of internet
   \( \mathcal{S} = \{ \text{real } t : t \geq 100 \text{ ms} \} \) infinite and continuous

5. Toss fair coin until first head
   \( \mathcal{S} = \{H, TH, TTH, \ldots\} \)
   \( = \{1, 2, 3, \ldots\} = \# \text{ of tosses needed} \)

Sample space can be finite or infinite
and discrete or continuous
An event is a subset of the sample space, $\Omega$, such that once I perform an experiment, it's either true or false.

I need 3 polls.

Event $A$ occurs if observed is an element of $A$ outcome.

1. Die tossing

   $A = \{2, 3, 5\} = \text{corresponds to rolling a prime}\#$

2. 2 Fair dice

   $A = \{(5, 6), (6, 5)\} = \text{rolling an eleven}$

3. Event $A = \{5, 7\}$ More than 5 polls are needed.

4. $A = \{t : 2 \leq t \leq 35 \}$

5. $A = \{H, TH, TTH\}$ Not more than 3 tosses are required.

Random event

**Set operation**

$A \cup B$ at least one of $A$ or $B$ occurs

$A \cap B$ both events $A$ and $B$ occur

$A^c$, complement of $A$ not $A$, i.e., $A$ does not occur

$\phi$ impossible event

$A \cap B = \phi$ Events $A$ and $B$ are mutually exclusive (cannot happen at same time)

$A \subset B$ $A$ implies $B$
Ω = the universe for a given random experiment
\\<\\\(\Omega = \emptyset\)
\\<\\\(\mathcal{P} = \Omega\)

Venn Diagram:

\[ A \cap B \neq \emptyset \]

\[ A \cup B = \emptyset \]

Tree diagram:

leaves are elements of \( \Omega \)

\( T_1, E_1, T_2 \)

\[ T_1 \cap T_2 \]

\[ E_1 \cap E_2 \]

How to measure probability?

Two problems:

1. Estimate likelihood of win
2. Divide up purse

Unless game is rigged, every elementary event is equally likely. Fair die:

6 \[ \cdots \cdot \cdot \cdot \]

36 points

Each outcome is equally likely.

Consider \( A = \{5, 6\}, \{6, 5\} \)

Can roll 1 eleven: \( \frac{1}{36} \)

\( P \{ A \} = \frac{n_A}{n_{total}} \)

2 eights: \( \frac{2}{26} \)
So, we produce a set of axioms to generalize the discrete case.

Consider a family of events defined on sample space $\Omega$.

**Properties:**

1. $\emptyset$, $\Omega$ are elements of $\mathcal{F}$.
2. If $A \in \mathcal{F}$, then $\overline{A} \in \mathcal{F}$.
3. If $A_1, A_2, \ldots \in \mathcal{F}$, the $\bigcup_{i=1}^\infty A_i \in \mathcal{F}$.

Define probability measure $P$ on $\Omega$ satisfying:

**Axioms:**

1. $P(A) \geq 0$, for all $A$ and is real-valued.
2. $P(\Omega) = 1$.
3. If $A, B \in \mathcal{F}$ and $A \cap B = \emptyset$ (mutually exclusive), then $P(A \cup B) = P(A) + P(B)$.
4. If $A_1, A_2, \ldots \in \mathcal{F}$ and $A_i \cap A_j = \emptyset$ for any $i$ and $j$, then $P\left(\bigcup_{n=1}^\infty A_n\right) = \sum_{n=1}^\infty P(A_n)$ for $i \neq j$.

Note that $A_1, A_2, \ldots$ are mutually exclusive.

$$P(\bigcup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$$

Consequence:

$P(\emptyset) = 0$.

**Proof:**

$A \cup \emptyset = A$ but $A \cap \emptyset = \emptyset$.

$\Rightarrow P(A \cup \emptyset) = P(A)$

$P(A) + P(\emptyset) = P(A)$

$P(\emptyset) = 0$. 

\[ P(A) = 1 - P(C \bar{A}). \]

Proof. \( A \cap \bar{A} = \emptyset \), \( \therefore P(C \cap A) = 1 \)

\[ A \cup \bar{A} = \Omega \]

\[ = P(C \cup A) \]

\[ = P(C \cap A) \]

\[ = P(C) + P(A) \]

What's the prob of \( \emptyset \)?

\[ A \cup B = (A \cap B) \cup (\bar{A} \cap B) \cup (A \cap \bar{B}), \]

\[ P(C \cup A \cup B) = P(C \cap A \cap B) + P(C \cap \bar{A} \cap B) + P(C \cap A \cap \bar{B}) \]

\[ (A \cap B) (A \cap \bar{B}) \Rightarrow P(C \cap A) = P(C \cap A) + P(C \cap \bar{B}) \]

mutually exclusive union is \( A \)

Similarly:

\[ P(C \cup B) = P(C \cap A \cap B) + P(C \cap \bar{A} \cap B) \]

\[ \therefore P(C \cup A \cup B) = P(C \cup A) + P(C \cup B) - P(C \cap A \cap B) \]

in general

If \( A \subset B \) then \( P(C \cap A) \leq P(C \cap B) \)

\[ B = A \cup (B - A) \] disjoint (= mutually exclusive)

\[ P(C \cap B) = P(C \cap A) + P(C \cap B - A) \]

\[ \Rightarrow 0 \]

\[ \therefore P(C \cap B) \geq P(C \cap A) \]

3 or 4 axioms \( \Rightarrow \) real prob of everything valued

1. Identify \( \Omega \), all elements must be mutually

2. Assign probabilities to elements in \( \Omega \)