Many areas of CE are characterized by lack of certainty. A probabilistic mechanism underlying.
- Not predictable except in some statistical sense.
- When do they occur? - phone system.
- How long will they be? - taxi cabs in city.
- Hospital beds.
- Website - what's capacity?

I. Idealized probabilistic model.
- Lists all possible outcomes and their probabilities.
- Any prediction must be validated in real life.
  - e.g., exit polls during political elections.
  - Draw conclusions based on a limited sample.
- Or, simulations.
- Consider benchmarking - typical configurations.

New operating system has improved error recovery.
- Provide assessment: how good it is.
- Error recovery (outcomes)
  - Successful recovery
  - Abortive error

Successful recovery happens with probability \( c \) (\( c \leq 1 \))
- Or abortive error = \( 1 - c \) probability
  - Note: \( c > 0 \).
$N$ errors
$n$ successfully recovered

$\frac{n}{N}$ is an estimate of the probability of successful recovery estimate.
Gets better as $N \to \infty$ i.e. # of observations

$$\lim_{N \to \infty} \text{Prob} \left\{ \left| \frac{n}{N} - p \right| > \epsilon \right\} = 0$$

Perform repeatedly
i.e. toss a fair coin repeatedly (repeated experiments)
expect a given outcome with a long-range frequency of $\frac{1}{2}$.

Intuitive view: tool that predicts when you repeat experiments.
Examples taken from queuing theory

<table>
<thead>
<tr>
<th>Event</th>
<th>customers</th>
<th>servers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\Rightarrow$</td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td></td>
</tr>
</tbody>
</table>

Event 1 = \{ arbitrary customer finds all servers busy $\}$
Event 2 = \{ arbitrary customer must wait $\geq x$ secs. for service $\}$
Event 3 = \{ # of waiting customers at arbitrary instant is $j$ $\}$

$A$, queue length = # of waiting customers at arbitrary instant
$N$, total number of customers in the $n$ servers system
$W$, waiting time of customer

Example 1 = \{ $N \geq s$ $\}$
$P\{N \geq s\}$ = probability of occurrence
Event 2 = \{W > x \} \quad P\{W > x \}

Event 3 = \{Q = j \} \quad P\{Q = j \}

Random variable: \( N, W, Q \)

device for describing numerical value for event

Coin tossing: only 2 outcomes: \( \{\text{head}\} \quad 1 \)
\( \{\text{tail}\} \quad 0 \)

\( X \) describes outcomes

\[ P\{\text{head tossed} \mid X = 1 \} \]
\[ P\{X = 0 \} \]

\( S_n = \# \text{heads in } n \text{ tosses} \)

\( X_j = \begin{cases} 
1 & \text{head} \\
0 & \text{tail}
\end{cases} \)

\[ S_n = \sum_{j=1}^{n} X_j \]

Homework due next Tuesday:

p. 23 \# 1, 2, 4, 6