Homework #2

p. 44 #1, 2

April 14/19 will be Midterm I

45 #14, 18

May 10/12 will be Midterm II

52 #11, 22

82 #2, 4

87 #1, 4, 8

97 #8, 12

106 #4, 9

119 #4, 8

Assumptions have to be in agreement w/ the problem.

Computer 5 peripherals # simultaneous active peripherals
Assess probability of coming across a bug
Exactly 1 peripheral is available

Step 1. Define the sample space

device \( \stackrel{<}{\rightarrow} \) busy

\( \overset{1}{\text{available}} \)

\( \left\{ 0, 0, 0, 0, 0 \right\} \)

set of all binary 5-tuples

25 points

\( p(\text{each}) = \frac{1}{32} \)

Step 2. Assign probs to the sample points
Assume each state is equally likely

Step 3. Identify the event of interest
Exactly 1 drive free:

\[ E = \{ S_1, S_2, S_4, S_8, S_{16} \} \]

Under the assumptions

\[ P(E) = \frac{5}{32} \text{ chance of trouble.} \]
Quality of software assessed

100 programs, events: S 1 I 0 SNI SNO I NO SINO

S, syntax bug

I, I/O issue

O, other (memory deallocation, ...)

If select program at random, what is prob (S or I or both)?

\[ P(SU-I) \]

\[ = P_S S + P_I I - P_S I \]

\[ = \frac{20}{100} + \frac{10}{100} - \frac{6}{100} \]

\[ = \frac{24}{100} \]

Write a performance guarantee

What is prob of a bug?

\[ P\{\text{error}\} = P\{S U I U O 0\} \]

\[ = P\{S U (I U O)\} = P_S S + P_I I U O - P_S I U O \]

\[ P\{S NI (I U O)\} = P\{S NI U (S N O)\} \]

\[ = P\{S NI I\} + P\{S NI O\} - P\{S NI NO\} \]

\[ = P_S S + P_I I - P_S I - P_S I NO - P_S S NI + P_S S NI \]

Note: Can only add probs of mutually exclusive events

![Diagram]
Combinatorics

With replacement: look at it and put it back
Without replacement: doesn't go back into pool of items

Permutation of order k = ordered selection of k items matters

Combination = order doesn't matter

\[ \text{pop} = \{x, y, z\} \text{ draw 2 letters} \]

Permutations w/ replacement: \(x, x, x, y, x, z, y, y, y, z, z, x, z, y, z\)
6 permutations w/o replacement: \(xy, xz, yx, yz, zx, zy\)

Combinations w/o replacement: \(xy, xz, yz\)

Multiplication principle:
Task A \(m\) different ways to do it
B \(n\)

\(A \text{ and } B\) together \(\Rightarrow mn\) different ways

Total of \(n\) items

Draw \(k\) items w/o repetition

How many different permutations there are?
\(n\) possibilities

\[
\frac{n!}{(n-k)!} = \frac{n!}{(n-k+1)!}
\]
How many different way of arranging $k$ items?

$k!$

A B C
a < b < c

$3 \times 2 \times 1 = 3!$

$\text{# of combinations } = \frac{n!}{(n-k)!k!}$

because order no longer matters

controller 5 devices
polled until I find device
ready to transmit

$\{0, 1, \ldots, 5\}$

$x_i =$ status of $i$th device
$0 =$ not ready
$1 =$ ready to transmit data

$\mathcal{S} = \{(x_1, x_2, x_3, x_4, x_5) \}$

25 sample points

# of ways to get (#of combinations

25 elementary events

Exactly 3 devices ready w/o replacement

$= \frac{5}{2!3!} = \binom{5}{3} = 10.$

$P\{\text{exactly 3 ready}\} = \frac{10}{32}$

Somebody now tells us exactly 3 ready, how many polls do I need to discover those 3, the first ready?
\[ A_1 \quad 1 \quad \binom{4}{2} = 6 \]

\[ A_2 \quad 2 \]

Event \( A_3 \): need 3 polls

\[ A_1: \quad \begin{array}{c}
0 \quad 0 \quad 0 \quad 0 \\
\uparrow \quad \text{ready}
\end{array}
\]

remaining 2 device is somewhere

here

\[ P(A_1 \text{ knowing } 3 \text{ are ready}) = \frac{6}{10} \]

\[ A_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \]

remaining 2 devices \( n_2 = \binom{3}{2} \)

\[ P(A_2 \text{ knowing 3 are ready}) = \frac{3}{10} \]

\[ A_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \]

only 1 configuration \( \binom{3}{1} \)

\[ P(A_3 \text{ knowing 3 are ready}) = \frac{1}{10} \]

\( A_1 \) or \( A_2 \) or \( A_3 \) are mutually exclusive.

Conditional probability

\[ \frac{P(A \text{ knowing } B \text{ occurs})}{P(B)} = \frac{P(A \cap B)}{P(B)} \]

\[ \text{Conditional probability} \]
Multiplication rule

\[ P(A \cap B) = P(A)P(B|A) \quad \text{if } P(A) \neq 0 \]
\[ = P(B)P(A|B) \quad \text{if } P(B) \neq 0 \]

\[ A_1, A_2, \ldots, A_n \]
\[ P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots \]
\[ \text{"AND"} \]
\[ P(A_n|A_1 \cap A_2 \cap \cdots \cap A_{n-1}) \]

Example
Market installation = 75%
100 installations
75 are customers
Select 3 installations

\[ P(\text{all are customers}) \]

\[ A_1, \ 1^{\text{st}} \text{installation is a customer} \]
\[ A_2, \ 2^{\text{nd}} \]
\[ \ldots \]
\[ A_n, \ n^{\text{th}} \]

\[ P(A_1 \cap A_2 \cap A_3) = \frac{75}{100} \times \frac{74}{99} \times \frac{73}{98} \approx 0.42 \]

Example
Memory
5000 chips total
Buy brandy (5% defect) 4000
Buy brand x (10% defect) 1000

\[ \text{event } A = \text{chip is of brand } x \]
\[ P(A) = \frac{1000}{5000} = 0.2 \]
\[ B = \text{chip is defective} \]
\[ P(B) = \frac{300}{5000} = 0.06 \]
\[ P(A \cap B) = \frac{100}{5000} = 0.02 \]

How likely that defective chip is X?
\[ P(E|B) = \frac{0.02}{0.06} = \frac{1}{3} = 0.33 \]
\[
\frac{P_{EAIB^3}}{P_{EBIA^2}} = \frac{P_{EAUB^3}}{P_{SA^3}} = \frac{P_{SA^3}}{P_{SB^3}}
\]