Thursday, Sept 30

HW #1 due Tuesday, at the beginning of class.

Once axioms followed, you get a deductive system.
Example of another deductive system: geometry in high school - axiomatic system

1. Identify the sample space, \( \Omega \).
   all elements must be mutually exclusive
   and collectively exhaustive
   must cover all possible outcomes
   cannot be further subdivided

2. Assign probabilities to elements in \( \Omega \).
   How likely every element of the sample space is
   based on elements from past experience
   or analysis of a random experience
   or assumptions

   Example,
   customer choices of an airline meal: will order
   \( 30\% \) pasta
   \( 70\% \) chicken known from past experience

   Example:
   die are fair = each face is equally likely
   \( P(\text{die}) = \frac{1}{6} \)
Example
New computer architecture
Make a simple assumption to make scientific workload
If you don't know
Make simple assumption

Example. # dots are not equal on 2 dice
How to assign prob space?

Example: \{0,1\}
How to assign prob?

3 Identify events of interest
In problem statement "How likely..."
you need to translate statement into some well-defined subset of \( \Omega \).
- Subset must be identifiable in \( \Omega \)
The question is answerable in sample space definition

4 Compute probabilities of events of interest.
Example
Imagine you are computer manufacturer
System can support 5 drives (devices) (peripherals)
What is prob: Exactly one drive (there is a bias/bug) available
Simultaneously active devices
5 drives

0. busy (state of drive
1. available)

State of system described: 5 4 3 2 1
5-tuple of 0's and 1's: \{0, 1, \ldots, 2^5\}

1. \[\bigcap S = \{ (x_1, x_2, x_3, x_4, x_5) : x_i = 0 \text{ or } 1, \ i=1, \ldots, 5 \}\]

2. Assume all elements are equally likely
   "uninformed prior"

3. Each element in $S$ has probability of $\frac{1}{32}$

4. "Exactly one drive is free" = event $E$
   corresponds to $E = \{ S_1, S_2, S_4, S_8, S_{16} \}$
   binary numbers
   $(1, 0, \ldots, 10)$
   $(0, 0, \ldots, 1)$
   Note: These events are mutually exclusive!

4. \[P(\text{bug showing up}) = P(E) = \frac{5}{32}\]

Example
Imagine a dept writing software
What's quality of software in that dept?
Run through a quality control program
Bugs:
1. syntax
2. I/O files not accessed properly
3. Other (memory corrupted, leaking, ...)
0
100 programs examined for bugs.

Event: \( S \land I \land O \land SNI \land SN0 \)

Probability: \( \frac{20}{100} \, \frac{10}{100} \, \frac{5}{100} \, \frac{6}{100} \, \frac{3}{100} \)

\( SNI \, SN0 \)

\( \frac{2}{100} \, \frac{1}{100} \)

Two questions:

1. Interested in prob of the event \( SNI \) program having syntax error or I/O or both? \( P(SNI, I, or both?) \)

2. What's prob that routine that dept wrote has an error? \( P(error) \)

1. \( P(SNI) = P(S3) + P(I3) - P(S3 \land I3) = \frac{24}{100} \)

2. \( P(error) = P(SNI \cup I0) \),

\( P(S3 \cup I0) = P(S3) + P(I0) - P(S3 \land I0) \)

\( P(S \land I0) = P((S \land I) \cup (S \land O)) \)

\( P(error) = P(S3) + P(I3) + P(O) + \cdots \)
\[ * = -P\{\text{IN0}\} - P\{\text{SNI}\} - P\{S \cap \text{NO}\} + P\{S \cup \text{NI} \cup \text{NO}\} \]

\[ A \quad B \]

Note: \[ P\{(S \cup I) \cup (S \cap NO)\} = P\{A \cup B\} - P\{A \cap B\} \]

\[ \therefore P\{\text{error}\} = \frac{9}{4} \]

**Combinatorics (chapter 2):** It's all about counting.

Finite sample space, \( S \)

Compute some event, \( E \)

\[ P(E) = \frac{\# \text{ of outcomes corresponding to } E}{\text{total } \# \text{ of outcomes}} \]

- Sets of elements
- Drawing samples from population
  - Imagine box of chips, checking if they have defects.
  - Two types of drawing
    1. With replacement = look at it & put it back in box
    2. Without replacement = look at it & Do NOT put back in box

**Two notions:**

1. **Permutation:** ordered selection
   
   Perm of order \( K \) = order matters. "ranked"

2. **Combination:** order does not matter
Example
Population = \{X, Y, Z\}

Drawing sample of 2 letters
Looking at permutations w/ replacement:
  xx, xy, xz, yx, yz, zy, zy, zz

Looking w/o replacement:
  6 permutations:
  xy, xz, yx, yz, zx, zy

Combinations w/o replacement:
  xy, yz, xz

\text{Multiplication principle:}
\text{task A accomplished in m different ways}
  \text{ B } \quad \text{ "} \quad \text{n}
  \text{ A & B } \quad \text{ "} \quad \text{mn"

Example
Population of n elements
Selecting k elements w/o replacement.

\text{How many permutations?} = n(n-1)(n-2)\ldots(n-k+1)
\frac{n!}{(n-k)!}
Example continued

In some ɛ0, permutations, define k elements in different positions. First ask:
(How many ways to arrange these?) k elements,

# of permutations for k elements
= k!

# of combination of k elements = \frac{n!}{k! \cdot \frac{n!}{(n-k)!}}
in population of n

real
Example

5 drives

\[ \begin{array}{cccc}
\mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\
\text{controller} & \text{drive or peripheral} \\
\end{array} \]

Will poll device to see if it is ready.

\[ x_i = \begin{cases} 
0 & \text{if drive is not ready (if needs service)} \\
1 & \text{if drive is ready} 
\end{cases} 
\]

for \( i = 1, \ldots, 5 \).

\[ \mathcal{A} = \left\{ (x_1, \ldots, x_5) \right\} \]

# of sample points = 32

How many sample points correspond to

Exactly 3 peripherals in ready state? 5 drives

\[ c' = \binom{5}{3} = \frac{5!}{(5-3)!3!} = 10. \]

i.e., have 5, select 3!
Assume we know 3 ready devices how many polls required to discover the 1st ready device?

count in this direction

4 drives

just need 1 poll


↓

need 2 polls

$A_1$: event 1 need 1 poll to discover 1st ready device

$A_2$: 2 polls

$A_3$: 3 polls

$A_1$: o o o o o

4 left

2 appear in those 4

$A_2$: o o o o o

select 2 out of 3

$A_2$: o o o o o

(3)

only have 1 choice

$P(S\text{ Needing 2 polls}) = \binom{4}{2} = \frac{4!}{(4-2)!2!} = \frac{6}{10} = 0.6$

$P(S\text{ Needing 3 polls}) = \binom{3}{2} = 3$
Conditional probability (or posterior)

\[ \Pr(A \mid B) \]

event A \hspace{1cm} \text{know 3 are ready}

event B \hspace{1cm} \text{Need one poll}

Define \[ \Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} \quad \text{if } \Pr(B) \neq 0. \]

= conditional probability of A given (knowing) B.