Today's quiz:

1. \[ \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} a^k b^{n-k} = ? \] Given \( a = 7 \), \( b = 3 \), \( n = 6 \)

2. \( q' = \frac{9}{5} \) \( g(t) = \) \# of animals (thousands) in years
   a) solve equation
   b) For \( t = 0 \), 1000 animals, find expression for \( g(t) \)
   c) After how many years, how more than 4000?

This quiz is graded but doesn't count towards your grade in this class.

Lecture:

- Phone system - how many lines? How to design the proper capacity?
- Cabs in a city? How many people will ask for a cab? How many licenses to issue?
- How many hospital beds to have?
- \# of applications supported? How to dimension system
- Idealized probabilistic model:
such that I give all possible outcomes and their probabilities.

Must calibrate with reality through limited data. Enter the realm of statistics.

2 possible outcomes: successful recovery or abortive error.

This happens with probability \( c \) and \( 1 - c \). Estimate \( c \).

Generate \( N \) errors. Look how many errors successfully recovered = \( n \).

As \( N \) increases, \( c \) tends to \( \frac{n}{N} \).

Formally, \( \lim_{N \to \infty} \text{Prob} \left\{ \left| \frac{n}{N} - c \right| > \varepsilon \right\} = 0 \)

means \( c \) “converges in probability” to \( \frac{n}{N} \).

Can never guarantee.
Coin, Throw it. Two possible outcomes
fair coin = 50% heads, 50% tails

Different possible outcomes have a long-run relative frequency that is constant.
means
in the long run, outcome is constant.

Example of events:

1. event is

   {an arbitrary customer finds all servers busy}

2. event = {an arbitrary customer must wait > x seconds for services}

3. event = {number of waiting customers at arbitrary instant is j}

\[ Q = \text{queue length} \]
\[ = \text{number of waiting customers} \]

\[ N = \text{number of customers in an s-server system} \]
(quad core \( s = 4 \))

\[ W = \text{waiting time of arbitrary customer until they get served} \]
rewrite event examples:

1. \( \mathbb{P}(N > s) \) \quad probability \ that \ \mathbb{N} \geq s \ of \ occurrence

2. \( \mathbb{P}(W > x) \)

3. \( \mathbb{P}(Q = j) \)

Capital letters are random variable is a device that represents events in terms of numerical values. \( X \) assigns numerical values to events; mapping of event to a value.

Coin tossing

\{ head \} \ \{ tail \} \ two \ possible \ events

\[
\begin{align*}
\text{head} & \rightarrow 1 \\
\text{tail} & \rightarrow 0 
\end{align*}
\]

toss coin \( n \) times

\( S_n = \) \# of heads in \( n \) tosses

\[
X_j = \begin{cases} 
1 & \text{head} \\
0 & \text{tail} 
\end{cases}
\]

\[
S_n = \sum_{j=1}^{n} X_j
\]