Quiz 2

Problem 1: Specify which of the following statements are true and which are false (no proof required):

1. If $A$ and $B$ are disjoint events such that $Pr(A), Pr(B) > 0$, then $Pr(A|B) = Pr(A)$. \textbf{False.}
2. If $S$ denotes the sample space and $A$ is any event in that space, then $Pr(A|S) = Pr(A)$. \textbf{True.}
3. If $A$ and $B$ are independent events, then $A^c$ and $B^c$ are independent events. \textbf{True.}
4. Suppose $B_1, B_2, \ldots, B_k$ is a partition of the sample space $S$ and let $F$ be an event in $S$, then

$$\sum_{i=1}^{k} Pr(B_i|F) = 1$$

\textbf{True.}

Problem 2: A machine produces defective parts with three different probabilities depending on its state of repair. If the machine is in good working order, it produces defective parts with probability 0.02. If it is wearing down it produces defective parts with probability 0.1. If it needs maintenance, it produces defective parts with probability 0.3. The probability that the machine is in good working order is 0.8, the probability that the machine is wearing down is 0.1 and the probability that it needs maintenance is 0.1.

1. Compute the probability that a randomly selected part will be defective. \textbf{3 pts}
2. Given that a randomly selected part is defective, what is the probability that the machine needs maintenance? \textbf{2 pts}

Solution:
Let $A_1$ be the event that machine is in working order, $A_2$ be the event that it is wearing down and $A_3$ be the event that it needs maintenance. Let $B$ be the event that the selected part is defective.

$$P(A_1) = 0.8$$
$$P(A_2) = 0.1$$
$$P(A_3) = 0.1$$
Also it is given that,

\[
P(B|A_1) = 0.02 \\
P(B|A_2) = 0.1 \\
P(B|A_3) = 0.3
\]

1. \[
P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) \\
= 0.02 \times 0.8 + 0.1 \times 0.1 + 0.3 \times 0.1 \\
= 0.056
\]

2. \[
P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)} \\
= \frac{0.3 \times 0.1}{0.056} \\
= 0.5357
\]

**Problem 3:** A, B and C are independent events with probabilities \(Pr(A) = 1/4, Pr(B) = 1/3\) and \(Pr(C) = 1/2\). Calculate the probability that A occurs but none of the other two events occur. \(3\) pts

**Solution:**

The required probability is \(P(AB^cC^c)\). Since A, B and C are independent, so are A, B^c and C^c. Hence

\[
P(AB^cC^c) = P(A)P(B^c)P(C^c) \\
= \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} \\
= \frac{1}{12}
\]