Problem 1: Suppose that a system contains three components that function independently of each other and are connected in series, that is, the system functions properly if and only if all components function properly.

Suppose that the lengths of the life of the components measured in hours follow exponential distributions with means 1,000; 3,000 and 6,000 respectively.

1. Determine the probability density function of the life length of the system. (5 points)

2. Determine the probability that the system will not fail before 100 hours. (2 points)

Problem 2: Suppose that in a large lot of manufactured items the probability of obtaining a defective item is 0.1. What is the smallest random sample of items that must be taken from the lot in order for the probability to be at least 0.99 that the proportion of defective items will be less than 0.13?

Hint: Use the CLT. (8 points)

Problem 3: The speed of a particle of mass $m$ is a random variable $v \sim N(0,1)$. The kinetic energy of the particle is a random variable $K = \frac{1}{2}mv^2$.

1. Find the expected value of $K$ (3 points)

2. Find the probability density function of $K$. (5 points)

Hint: Remember that $\Phi'(x) = \phi(x)$.

Problem 4: Suppose that the number of customers that shop at a given supermarket in one day is a random variable with Poisson distribution with mean 100. Suppose that the amount spent by each customer is a random variable, independent of the number of customers, that follows a normal distribution with mean $\$50$ and standard deviation $\$24$. Then, the total amount spent by all customers in that supermarket in one day, say $Y$, is the sum of the quantities spent by each customer. Such a sum has a random number of terms. Find the expected value of $Y$.

Hint: Use conditional expectations. (5 points)

Problem 5: The random variables $X$ and $Y$ have joint density function given by

$$f(x, y) = \begin{cases} \frac{2e^{-2x}}{x} & 0 < x < \infty; 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Compute $\text{cov}(X, Y)$. (7 points)