Homework 5

Reading: *Book of Proof* Sections 5.1-5.3 and 6.1-6.4

1 point will be awarded if there is a staple keeping all of the pages of your homework together, even if there is only one page.

Problems: Due Wednesday October 28, 2015 9:30am PST in class. Use other sheets. There is not enough room for correct answers here. Show work!

1. (5 points) Calculate the value of the following expressions. Show work!!
   (a) \( \gcd(144, 180) \)
   (b) \( \text{lcm}(144, 180) \)
   (c) \( \gcd(1276, 174) \)
   (d) \( \text{lcm}(1276, 174) \)
   (e) \( \gcd(308, 286) \cdot \text{lcm}(308, 286) \)

2. (5 points) Prove the following theorem:
   **Theorem** For any prime number \( p \) and natural numbers \( a \) and \( b \), if \( p \nmid b \) then \( ap \nmid b \).

3. (5 points) Prove the following theorem:
   **Theorem** For any integers \( n \) and \( m \), \( n^2 - 8m \neq 2 \).

4. (5 points) Prove the following theorem:
   **Theorem** For any integer \( n \), if \( 7n^2 + 22n + 3 \) is odd, then \( n \) is even.