Homework 1

Reading: Book of Proof Sections 1.1-1.4

1 point will be awarded if there is a staple keeping all of the pages of your homework together, even if there is only one page.

Problems: Due Wednesday September 30, 2015 9:30am PDT in class.
There is not enough room for correct answers here. Show work.

Here $\emptyset$ is the empty set, $\mathbb{N}$ is the set of natural numbers, $\mathbb{Z}$ is the set of integers, and $\mathbb{R}$ is the set of real numbers

1. (8 points) Express each of the sets below by listing all of their elements when the set is finite, and at least 6 elements when the set is infinite. For example, $\{4n-1 \mid n \in \mathbb{Z}\}$ while $\{5,9,13,21,25,29\}$ is the answer for $\{4n+1 \mid n \in \mathbb{N} \text{ and } n < 8\}$.
   
   (a) $\{n \in \mathbb{Z} : 0 < n^2 - 1 < 48\}$
   (b) $\{\cos \frac{n\pi}{2} : n \in \mathbb{Z}\}$
   (c) $\{7 - 3n : n \in \mathbb{N}\}$
   (d) $\{n : 7 - 3n \in \mathbb{N} \text{ and } 7 - 3n \leq 8\}$
   (e) $\{-11,7\} \times \{1,2\} \times \mathbb{R}$
   (f) $\{1,2,3\} \times \{a,b,c,d,e,f\} \times \emptyset$
   (g) $\{p : p \text{ was elected president of the United States exactly twice in the 20th century}\}$
   (You might want to use Google or some other search engine for this problem.)
   (h) $\{X \subseteq \{3,4\} \times \{1,2\} : |X| = 3\}$

2. (10 points) For each of the sets below give the size of the set if it is finite, and otherwise state that it is infinite.
   (a) $\{1,2,3,7\}$
   (b) $\emptyset$
   (c) $\{\emptyset\}$
   (d) $\{\emptyset,\emptyset,\emptyset\}$
   (e) $\{\{1\},\{\{2\},3,4\},23,\{4,\pi\}\}$
   (f) $\{\mathbb{N},\emptyset,\mathbb{Z}\}$
   (g) $\{1,2,3,4\} \times \{5,6\} \times \{7,9,10,11\}$
   (h) $\mathbb{Z} \times \{6\}$
   (i) $\mathbb{N} \times \emptyset$
   (j) The power set of $\{a,b,c,d,e,f\}$

3. (10 points) For this problem let
   
   $A = \{4k-3 : k \in \mathbb{Z}\}$
   $B = \{n^2 - 7 : n \in \mathbb{N} \text{ and } n \text{ is even}\}$
   $C = \{m : m \in \mathbb{Z} \text{ and } -10 \leq m \leq 10\}$.

   State whether each of the statements below is True or False and justify your answer. $X \subseteq Y$ means $X$ is a proper subset of $Y$; $X$ is a subset of $Y$ and $X \neq Y$.

   (a) $C \subseteq \mathbb{N}$
(b) \( \mathbb{N} \subseteq C \)
(c) \( A \subseteq B \)
(d) \( B \subseteq A \)
(e) \( B \subseteq \mathbb{Z} \)

4. **(4 points)** Give the power set of each of the sets below.

(a) \{1, 2, 3\}
(b) \( \emptyset \)
(c) \( \{\mathbb{R}\} \)
(d) \( \{2, \{3\}\} \)