1. What is the value of the sum $\sum_{i=1}^{n} (3^i - 5i + 20)$ (20 points)

\[
\sum_{i=1}^{n} (3^i - 5i + 20) \\
= \sum_{i=1}^{n} (3^i) + \sum_{i=1}^{n} (-5i) + \sum_{i=1}^{n} (20) \\
= \frac{3^{n+1} - 1}{3 - 1} - 5 \frac{(n+1)n}{2} + 20n \\
= \frac{3^{n+1} - 3/2}{2} - \frac{5n^2 + 5n}{2} + 40n \\
= \frac{3^{n+1} - 5n^2 + 35n - 3}{2}
\]

2. Use mathematical induction to prove $\sum_{i=1}^{n} (8i + 2) = 4n^2 + 6n$ (20 points)

Basic Step: The equation is true for $n=1$ because the left side is 10 and the right side is 4+6=10

Induction Step: If $P(k)$ is true, that means $\sum_{i=1}^{k} (8i + 2) = 4k^2 + 6k$, we need to prove $P(k+1)$ is true, that means $\sum_{i=1}^{k+1} (8i + 2) = 4(k + 1)^2 + 6(k + 1)$

Notice that for the left side, the difference between $P(k)$ and $P(k+1)$ is $8(k+1)+2=8k+10$

For the right side, the difference between $P(k)$ and $P(k+1)$ is $4(k + 1)^2 + 6(k + 1) - 4k^2 - 6k$

The differences between $P(k)$ and $P(k+1)$ on both sides are the same, therefore, if $P(k)$ is true, $P(k+1)$ is also true.
3. Use mathematical induction to prove $99n < n^3$ for integer $n \geq 10$ (20 points)

Basic step: for $n=10$, $990 < 1000$, therefore true.
Induction step: The difference between $P(k)$ and $P(k+1)$ for the left side is $99(k+1) - 99k = 99$

The difference between $P(k)$ and $P(k+1)$ on the right side is $(k+1)^3 - k^3 = 3k^2 + 3k + 1$
This is larger than 99 when $n \geq 10$.
Therefore, if $P(k)$ is true, then $P(k+1)$ is also true.

4. Use mathematical induction to prove $3|4^n - 1$ (20 points)

Basic step: $3|4^1 - 1$ is true
Induction step: if we assume $3|4^k - 1$, we want to prove that $3|4^{k+1} - 1$
Notice that the difference between $4^{k+1} - 1$ and $4^k - 1$ is $3 \cdot 4^k$, which is divisible by 3.
Therefore, if $3|4^k - 1$, then $3|4^{k+1} - 1$. 