1. Suppose the domain of $x$ includes all the teapots in a kitchen and the domain of $y$ includes all the lids in the kitchen. Suppose the predicate $P(x,y)$ means a lid $y$ fits a teapot $x$.

(a) Translate the following sentence into a proposition using quantifiers, predicate, and logical operators: “If there is a lid that fits all teapots then for every teapot there exists a lid that fits it” (10 points).

$$(\exists y \forall x P(x,y)) \rightarrow (\forall x \exists y P(x,y))$$

(b) Translate the following proposition into an English sentence:

$$(\forall y \exists x \exists z (x \neq z \land P(x,y) \land P(z,y)))$$

Every lid fits at least two teapots
2. What is the truth value of the statement “∀x∃y (x⋅(y+0.1)=0)” if the domains of x and y are real numbers, please explain. (truth value: 3 points; explanation: 7 points) What if the domains of x and y are integers, please explain. (truth value: 3 points; explanation: 7 points)

When the domains are real numbers, it is true. Because for every x, if y=-0.1, then it is true.
When the domain are integers, it is false. Because for example, for x=1, no matter what y is, y+0.1 will not be 0, therefore, x(y+0.1) will not be 0.

3. Let A={1,2,3,4} and B={x| x is an integer and 2 ≤ x ≤ 6}. Find (a) A∪B (b) A∩B (c) A-B (d) All the subsets of A. (20 points, 5 points each)

Solution:
(a) A∪B = {1,2,3,4,5,6}
(b) A∩B = {2,3,4}
(c) A-B = {1}
(d) All the subsets of A:
∅,
{1}, {2}, {3}, {4},
{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4},
{2,3,4}, {1,3,4}, {1,2,4}, {1,2,3},
{1,2,3,4}
4. Let A, B, and C be sets. Use Venn diagram to show that \((B - A) \cap (A - C) = \emptyset\). Please explain. (15 points, Venn diagram 10 points, explanation 5 points)

Solution:

The Venn diagram: