Chapter 1.

Logic and proof: Section 1.1-1.5

Proposition, Operations of propositions ($\neg, \land, \lor, \rightarrow, \leftrightarrow, \oplus$), negation, and, or, implication, biconditional, exclusive or): *Translation English to proposition and vice versa*

Truth table: *Truth table of proposition*

Tautology, contradiction, contingency: *Determine contradictions and tautology*

Logically equivalent: $p \leftrightarrow q$ is a tautology, to verify this, try all truth values for the variable in $p$ and $q$ and show that $p$ and $q$ has the same truth values

Some logical equivalences: Laws in 1.2 Table 5: *Use existing equivalences to prove other equivalences*

Predicate – prepositional function, the truth value is determined after the truth values of all variables are given.

Quantifier: universe of discourse, existential quantifier $\exists$ and universal quantifier $\forall$.

*Translate English to (nested) quantifications and vice versa*

*Determine the truth values of (nested) quantifications*

Rules of inference: 1.5 Table 1

*Translate English into proposition, derive conclusions using rules of inference.*

Methods of Proving Theorem

Direct proof, indirect proof, proof by contradiction, proof by cases

Sets: Section 1.6-1.7

Set, set builder, empty set (Null set), Venn diagram, subset, proper subset, Cartesian product

Set operations: union, intersection, difference, complement, Set identities (1.5 Table 1)

*Set builder and basic concepts and operators*

*Use Venn diagram to prove equality*

Functions: Section 1.8

Function, domain, image, one-to-one, onto, bijection/one-to-one correspondence, inverse function, floor function, ceiling function

*Determine if a function is onto, one-to-one, and bijection*

Chapter 2.
2.4 Division: $a|b$, divisibility, primes and composite, verify a prime, dividend, divisor, quotient, residual, gcd, pairwise primes, least common multiple, modular arithmetic, congruent

Prime factorization, gcd, lcm using factorization;

2.5 (binary, octal, hexadecimal) $\leftrightarrow$ decimal, binary operation
Euclidean algorithm for finding gcd

Chapter 3.
3.1 Backward reasoning, counterexample, proof by construction

3.2 Sequence and Summations: solve ($\sum_{i=m}^{n} ai^2 + bi + c$, $\sum_{i=m}^{n} ar^{i} + b$, $\sum_{i=1}^{m} \sum_{j=1}^{n} (ai + br^{i} + cj + dr^{j} + e)$)

3.3 Mathematical induction
Step 1: What is the proposition $P(n)$
Step 2: Basis step: show that $P(0)$ is true
Step 3: Inductive step: show that $P(n+1)$ is true if $P(n)$ is true
Strong mathematical Induction.

Four types of problem: summation of polynomials or powers, divisibility and modular arithmetic, inequality, sets

3.4 Recursive definitions
The basis value and the inductive definition,
Evaluation and derivation of recursive definitions.
Recursive definition $\leftrightarrow$ normal function definition

Chapter 4,5.
4.1 Counting basics
The product rule, the sum rule, combo

4.3 Permutation and combinations
Formulae for permutation and combination, $c(n+1, r) = c(n, r) + c(n, r-1)$, Pascal’s Identity

Problems: cards, passwords, bit strings, tennis team members, etc

5.1 Discrete probability
Solving two counting problems: $|E|$ and $|S|$, use permutation or combinations on both

Problems: cards, passwords, bit strings, tennis team members, etc

4.2 Pigeon hole principle
The generalized principle: k objects, m bins, there are at least ceiling(k/m) objects in one of the bins.

Two types problem: (1) known k, m, find how many objects in one bin
(2) known ceiling(k/m) and m, how many objects are required.

Chapter 6.
6.1 Recurrence relations
Rabits and the Fibonacci numbers, the tower of Hanoi, compound interest

6.2 Solving recurrence relations
Roots of characteristic equation, linear combination of the powers of the roots, solving for the linear coefficients from the initial condition