CMPE 8  
NAME: **In class solutions**  
Fall 2009 Midterm

Test Time: 8:05–8:35 AM

1. (5 points) Write the Matlab command to generate a vector \( t \) that starts with 4, ends with 10, and has a sample period of 0.2.

\[
\begin{array}{c}
t = [4; 0.2; 10]
\end{array}
\]

2. (6 points) For the time vector \( t \) defined above, what are: a) \( t(2:4) \), b) \( \text{length}(t) - t(\text{end}) \), and c) \( t(\text{end})-1 \)? Write the numeric answers.

a) \( [4.2, 4.4, 4.6] \)

b) \( 31 - 10 = 21 \)

c) \( 10 - 1 = 9 \)

d) **Bonus**: (3 points) Write the Matlab commands to subtract the first 10 elements from the last 10 elements for the time vector \( t \), and raise the resulting vector to the power 4.

\[
\begin{array}{c}
( t(22:31) - t(1:10) ) \cdot 4
\end{array}
\]

3. (3 points) Given a dynamic model \( x_{k+1} = f(x_k) \), for \( k = 1, 2, ..., \) and a start value \( x_1 \), define what the **orbit** starting from \( x_1 \) is for this model, using words and equations as necessary.

4. (3 points) Given a dynamic model \( x_{k+1} = f(x_k) \), define what a **fixed point** (also known as an **equilibrium point**) is, using words and equations as necessary.

5. Consider the model

\[
x_{k+1} = \frac{3}{4} x_k + x_k^3
\]

(a) (6 points) Calculate the fixed point(s) \( x_* \).

\[
\begin{array}{c}
x_* = \frac{3}{4} x_* + x_*^3 \\
-x_* = -x_* \\
0 = -\frac{3}{4} x_* + x_*^3 = x_* (x_*^2 - \frac{1}{4})
\end{array}
\]

\[
x_* = 0, \frac{1}{2}, -\frac{1}{2}
\]
(b) (5 points) Compute the four orbits \((x_1, \ldots, x_6)\) starting from \(x_1 = +0.3, -0.4, +0.7,\) and \(-0.6\).

orbit 1:  
\[
\begin{pmatrix} 0.3 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}
\]
approach 0

orbit 2:  
\[
\begin{pmatrix} -0.4 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}
\]

orbit 3:  
\[
\begin{pmatrix} +0.7 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}
\]
\(\pm 100,000\) diverge

orbit 4:  
\[
\begin{pmatrix} -0.6 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}
\]

(c) (5 points) Based on the orbits calculated, determine the stability of each fixed point \(x_*\) (stable, stable and attractive, or unstable).

\(x_* = 0 - \frac{3}{4} a\) - unstable

\(x_* = \pm \frac{1}{2}\) - unstable

6. **Bonus**: (5 points) Reconsider the model above now as

\[x_{k+1} = \frac{3}{4} x_k + x_k^3 + u_k\]

with “feedback control” \(u_k = -x_k^3 + ax_k\). The control objective is to “linearize the system” and make the origin \((x_* = 0)\) the only fixed point and to make it stable and attractive.

For what value or values of \(a\) does the controller meet this objective?

\[x_{k+1} = \left(\frac{3}{4} + a\right) x_k\]

Model is now in linear form. \(a \neq \frac{1}{4}\)

Recall properties of linear models...

For coefficients \(\neq 1\), the only fixed point is \(x_* = 0\)

For coefficients between \(-1\) and \(1\), the fixed point will be stable and attractive.

Let \(\left|\frac{3}{4} + a\right| < 1\) \(\Rightarrow\) \(-\frac{7}{4} < a < \frac{1}{4}\)