Given $x_{n+1} = F(x_n)$
(for example $x_{n+1} = \cos(x_n)$
OR
$x_{n+1} = x_n^2 - 1$)

With any value $x_1$, you can compute (with MATLAB or calculator) the trajectory $(x_1, x_2, x_3, ...).$
The start value is called the "initial condition." $\text{IC}$ abbreviation

Example: Model: $x_{n+1} = x_n^2 - 1$

$\text{IC: } x_1 = 0$

Trajectory: $(0, -1, 0, -1, 0, -1, ...)$

$x_2 = x_1^2 - 1 = 0^2 - 1 = -1$

Examples of "oscillation"
IC: \[ x_1 = 2 \]

Traj.: \( (2, 3, 8, 63, \ldots) \)

Example of "diverging"

IC: \[ x_1 = 1 \]

Traj.: \( (1, 0, -1, 0, \frac{1}{4}, 0, -1, \ldots) \)

A start value \( x_1 \) is called a fixed point (a.k.a. equilibrium point) if \( x_1 = f(x_1) \). We denote this value as \( x^* \).

Task 3.3.2 \( f(x) = x + x^2 - 1 \)

Step 1: Write the model

\[ x_{n+1} = f(x_n) = x_n + x_n^2 - 1 \]
Step 2: Replace $X_{n+1}$ with $X_n$

\[ X_n = X_n^2 + x_n^2 - 1 \]

Better: Replace $X_{n+1} \frac{1}{2} X_n$ with $X*$

\[ X* = X_* + x_*^2 - 1 \]

Step 3: Solving for $X*$

Suggestions:

- Don't divide both sides by $X*$

E.g. \( (x_* - X_*) < 0 \)

Get \( x_* - 1 = 0 \)

\[ X_* = 1 \checkmark \]

- Factor out instead

\[ X_* (x_* - 1) = 0 \]

\[ \Rightarrow X_* = 0, 1 \]

\[ \Rightarrow x_*^2 = 1 \Rightarrow x_* = \pm 1, -1 \]
Group Task 3.3.3

No eq. pts. \( x_{n+1} = e^{x_n} \)
\[ x_{n+1} = x_n^2 \quad (x_0 = 0, 1) \]
\[ \rightarrow x_{n+1} = x_n + b \quad b \neq 0 \]

Infinitely many eq. pts.
\[ \rightarrow x_{n+1} = x_n \]
\[ x_{n+1} = c \quad (x_0 = c) \]
\[ x_{n+1} = \sin(x_n) + x_n \]
\[ \sin(x_0) = 0 \]
\[ \Rightarrow x_0 = 0, \pm \pi, \pm 2\pi, \ldots \]

Group Task 3.3.4

Flat ground: \( h_k = 0 \)
\[ v_{k+1} = v_k + \frac{\Delta}{m} [-b v_k] \]
Equilibrium \( \Rightarrow \)
\[ v_* = v_* + \frac{\Delta}{m} [-b v_*] \]
Common form in mechanics

\[ x_{k+1} = f(x_k) = x_k + g(x_k) \]

equilibrium \( \Rightarrow g(x^*) = 0 \)

OR \( x_{k+1} = x_k + m \frac{\Delta g(x_k)}{\Delta t} \)

Sample period

\[ m \frac{x_{k+1} - x_k}{\Delta} = g(x_k) \lim_{\Delta \to 0} \frac{\max_{x} \Delta t}{\Delta t} \]

GT 3.3.4

Cont'd \( \frac{\Delta}{m} (\cdot b) \cdot V^* = 0 \)

\( \neq 0 \Rightarrow V^* = 0 \)

\[ \frac{1}{m} \frac{\Delta}{m} (-b) \cdot V^* = 0 \]

\[ V_{k+1} = V_k + \frac{\Delta}{m} \left[ -bV_k + h_k \right] \]

\[ V^* = \frac{h_k}{b} \]