DIGITAL LOGIC: FROM TRANSISTORS TO GATES

CE1 - Hands-on Computer Engineering

Graded

HW 1 on back desk

HW 2 due to me
LAST WEEK:
Bread Board
**Resistor Color Code**

- Black
- Brown
- Red
- Orange
- Yellow
- Green
- Blue
- Purple
- Grey
- White
- Gold
- Silver

```
162K Ohm ± 1%
```
**This week: Logic!**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>$F(A,B)$</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>GND</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$V_{cc}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$V_{cc}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$V_{cc}$</td>
</tr>
</tbody>
</table>
This week: Chips!

7400
Transistor: building block of computers
Microprocessors contain tons of transistors
- AMD 6-core Opteron (2009): 904 million
- Intel Core i7 Quad (2008): 731 million
- Intel Core 2 Duo (2006): 291 million
- Intel Itanium 2 (2003): 220 million
- Intel Pentium 4 (2000): 42 million
- IBM PowerPC 750FX (2002): 38 million
- Intel 4004 (1971): 2300
THE TRANSISTOR: PAST AND PRESENT
“The number of active components per chip will double every 18 months.”
GPU Processing Compared to CPU

<table>
<thead>
<tr>
<th>GFLOPS</th>
<th>G80GL = Quadro 5600 FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>G80 = GeForce 8800 GTX</td>
</tr>
<tr>
<td>200</td>
<td>G71 = GeForce 7900 GTX</td>
</tr>
<tr>
<td>100</td>
<td>G70 = GeForce 7800 GTX</td>
</tr>
<tr>
<td>0</td>
<td>NV40 = GeForce 6800 Ultra</td>
</tr>
<tr>
<td>0</td>
<td>NV35 = GeForce FX 5950 Ultra</td>
</tr>
<tr>
<td>0</td>
<td>NV30 = GeForce FX 5800</td>
</tr>
</tbody>
</table>

- G80GL = Quadro 5600 FX
- G80 = GeForce 8800 GTX
- G71 = GeForce 7900 GTX
- G70 = GeForce 7800 GTX
- NV40 = GeForce 6800 Ultra
- NV35 = GeForce FX 5950 Ultra
- NV30 = GeForce FX 5800

Timeline:
- January 2003: NV30
- June 2003: NV35
- April 2004: NV40
- May 2005: G70
- G70-512
- November 2005: G71
- March 2006: G80
- November 2006: G80GL

Intel Core2 Duo: 3.0 GHz
What Is a Transistor?

- A switch, which can close between the source and the drain
- Changing the voltage of the gate lets you change the current flow between the source and drain (closing or opening the switch)
METAL-OXIDE-SEMICONDUCTOR TRANSISTOR

NMOS Transistor (n-channel MOSFET)

Silicon Dioxide (insulator)
metal tracks
source
gate
drain
gate electrode
p-type silicon
n-channel
n-type silicon
Silicon Substrate
How big is a transistor?

- If a CPU die were as big as this whole classroom...
- A transistor would be...

Core 2 duo: 291 million transistors
143 sq mm
3 or 4 transistors would fit on the end of a ball point pen (1 sq mm).
WHAT IS A TRANSISTOR?

- Logically, each transistor is used as a switch
- Combined to implement logic functions
  - AND, OR, NOT
- Combined to build higher-level structures
  - Adder, multiplexer, decoder, register, ...
- Combined to build a processor
  - Pentium, Core 2 Duo, etc
**Simple switch circuit**

*Switch-based circuits* can easily represent two states: on/off, open/closed, voltage/no voltage.

**Switch open:**
- No current through circuit
- Light is off
- $V_{\text{out}}$ is $+2.9V$

**Switch closed:**
- Short circuit across switch
- Current flows
- Light is on
- $V_{\text{out}}$ is $0V$
N-type MOS Transistor

**n-type MOS (nMOS)**

- when Gate has **positive** voltage, short circuit between #1 and #2 (switch closed)
- when Gate has **zero** voltage, open circuit between #1 and #2 (switch open)

Terminal #2 must be connected to GND (0V).
**P-type MOS transistor**

**p-type** is complementary to n-type

- when Gate has **positive** voltage, open circuit between #1 and #2 (switch open)
- when Gate has **zero** voltage, short circuit between #1 and #2 (switch closed)

Terminal #1 must be connected to +2.9V in this example.
Digital Values for Analog Signals

- Use the switch behavior of MOS transistors to implement logical functions: AND, OR, NOT
- Digital symbols:
  - We assign a range of analog voltages to each digital (logic) symbol
  - Assignment of voltage ranges depends on electrical properties of transistors being used

<table>
<thead>
<tr>
<th>Digital Values</th>
<th>&quot;0&quot;</th>
<th>Illegal</th>
<th>&quot;1&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Values</td>
<td>0</td>
<td>0.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>
CMOS CIRCUIT

- CMOS is Complementary Metal Oxide Semiconductor
- Uses both n-type and p-type MOS transistors
  - p-type (pMOS)
    - Attached to + voltage
    - Pulls output voltage UP when input is zero
  - n-type (nMOS)
    - Attached to GND
    - Pulls output voltage DOWN when input is one
The most basic representation of a logic function

It is a perfect induction proof - Lists the output for all possible input combinations

How many rows of the truth table needed?
**Truth Table: Inverter**

- Inverted signals are denoted with an overbar
- Or with a prime symbol
  - $A'$

Inverted signals are denoted with an overbar or with a prime symbol.

### Input

<table>
<thead>
<tr>
<th>A</th>
<th>Y = A'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
**Inverter (NOT Gate)**

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 V</td>
<td>2.9 V</td>
</tr>
<tr>
<td>2.9 V</td>
<td>0 V</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[2.9V\]

\[6\text{nd}\]

\[P\text{-type}\]

\[N\text{-type}\]

\[\text{In}=0\]

\[\text{Out}=1\]

\[\text{In}=1\]

\[\text{Out}=0\]
What makes gates go: transistors as switches

<table>
<thead>
<tr>
<th>IN</th>
<th>P-type</th>
<th>N-type</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Closed</td>
<td>Open</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Open</td>
<td>Closed</td>
<td>0</td>
</tr>
</tbody>
</table>
**Truth Table: AND Gate**

- The result of an AND operation is 1 if and only if all inputs are 1.
- Depict AND by the multiplication symbol $A \cdot B$.
- Or by lumping the signals together as $AB$.
- We don’t really build these gates...

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B</td>
<td>Y = A · B</td>
</tr>
<tr>
<td>0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1</td>
<td>0</td>
</tr>
<tr>
<td>1 0</td>
<td>0</td>
</tr>
<tr>
<td>1 1</td>
<td>1</td>
</tr>
</tbody>
</table>
WHAT MAKES NAND GATES GO: TRANSISTORS AS SWITCHES

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Add an inverter to a NAND.
**Truth Table: OR Gate**

- The result of an OR operation is 1 if and only if any inputs are 1.
- Depict OR by the addition symbol $A + B$.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B</td>
<td>Y = A + B</td>
</tr>
<tr>
<td>0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1</td>
<td>1</td>
</tr>
<tr>
<td>1 0</td>
<td>1</td>
</tr>
<tr>
<td>1 1</td>
<td>1</td>
</tr>
</tbody>
</table>
NOR GATE: NOT-OR

Note: Serial structure on top, parallel on bottom.
Add an inverter to a NOR gate.
BUILDING LOGIC FOR A TRUTH TABLE

- Simple logic functions can be expresses in a truth table form
  - Why would we not want to use truth tables for large number of input functions?
- To implement this function in logic we can use the basic gates AND, OR, and NOT
- One method to do this is called “Sum of Products” another is “Product of Sums”
- There are other methods, including ways to reduce the size of the logic
## Sum of Products

- **XOR Gate**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>Y</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Find the rows with the ‘1’ output
2. Write the product-form expression for the inputs in that row (0=inverted, 1=normal)
3. Combine the products in step 2 into a sum (OR the results of step 2)

\[ Y = \overline{A} \cdot \overline{B} + A \cdot \overline{B} \]
**PRODUCT OF SUMS**

- **Procedure:**
  1. Find the rows with the ‘0’ output
  2. Write the sum-form expression for the inputs in that row (0=normal, 1=inverted)
  3. Combine the sums in step 2 into a product (AND the results of step 2)

- **Note:** we treat 0 and 1 reverse than for SoP

\[ \gamma = (A + B). (\overline{A} + \overline{B}) \]
**Logical Completeness**

Can implement **ANY** truth table with AND, OR, NOT.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
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<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Sum of Products Method:**

1. AND combinations that yield a "1" in the truth table.
2. OR the results of the AND gates.
3. Is not necessarily a minimal solution.
USING A CHIP:
CONNECTING YOUR CHIP TO POWER AND GROUND
Now you can connect the inputs to one of the gates to either power or ground.
Now you can connect the output to an LED so you can verify the truth table.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
USEFUL CHIP DIAGRAMS
Simple Electronics: Ring Oscillator

Diagram of a ring oscillator with labeled input and output signals.
Resistor (Ohms)

Capacitor (Farads)

4.7MΩ

0.1μF
Light Emitting Diode (LED)