1. Let $y_i$, $i = 1, ..., n$, be realizations of independent random variables $Y_i$ following gamma($\mu_i, \nu$) distributions, with densities given by

$$f(y_i; \mu_i, \nu) = \frac{(\nu/\mu_i)^\nu y_i^{\nu-1} \exp(-\nu y_i/\mu_i)}{\Gamma(\nu)}, \quad y_i > 0; \quad \nu > 0, \mu_i > 0,$$

where $\Gamma(\nu) = \int_0^\infty t^{\nu-1} \exp(-t)dt$ is the Gamma function.

(a) Express the gamma distribution as a member of the exponential dispersion family.

(b) Obtain the scaled deviance and the deviance for the comparison of the full model, which includes a different $\mu_i$ for each $y_i$, with a gamma glm based on link function $g(\mu_i) = x_i^T \beta$, where $\beta = (\beta_1, ..., \beta_p)$ ($p < n$) is the vector of regression coefficients corresponding to a set of $p$ covariates.

2. Consider the data set from:


on the incidence of faults in the manufacturing of rolls of fabric. The first column contains the length of each roll (the covariate with values $x_i$), and the second contains the number of faults (the response with means $\mu_i$).

(a) Use R to fit a Poisson regression model, with a logarithmic link,

$$\log(\mu_i) = \beta_1 + \beta_2 x_i$$

(1)

to explain the number of faults in terms of length of roll.

(b) Fit the regression model for the response means in (1) using the quasi-likelihood estimation method, which allows for a dispersion parameter in the response variance function. (Use the quasipoisson “family” in R.) Discuss the results.

(c) Derive point estimates and asymptotic interval estimates for the linear predictor, $\eta_0 = \beta_1 + \beta_2 x_0$, at a new value $x_0$ for length of roll, under the standard (likelihood) estimation method from part (a) and also under the quasi-likelihood estimation method from part (b). Evaluate the point and interval estimates at $x_0 = 500$ and $x_0 = 995$.

(Under both cases, use the asymptotic bivariate normality of the vector $(\hat{\beta}_1, \hat{\beta}_2)$ to obtain the asymptotic distribution of $\hat{\eta}_0 = \hat{\beta}_1 + \hat{\beta}_2 x_0$.)