Problem 1 Consider the following linear system and show that
\[
\begin{aligned}
\dot{x}_1 &= x_2 + x_3 \\
\dot{x}_2 &= 2x_3 + 2x_2 \\
\dot{x}_3 &= u
\end{aligned}
\]

1. there is no linear feedback control, \( u = kx \), so that the origin is asymptotically stable; (use rank condition: \( \text{rank}[B, AB, A^2B, \ldots, A^{n-1}B] = n \))

2. there is no continuous feedback, \( u = k(x) \), so that the origin is asymptotically stable. (use Brockett’s necessary condition)

Problem 2 Design an observer for the following system and show that the estimation error converges to zero.
\[
\begin{aligned}
\dot{x}_1 &= x_2 - x_1^3 \\
\dot{x}_2 &= x_1 - 2x_2 + \sin t \\
y &= x_1
\end{aligned}
\]

Problem 3 Design an output feedback control for the following system so that the origin is globally asymptotically stable.
\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u + x_2 \sin x_2 \\
y &= x_1
\end{aligned}
\]
(No need to compute the exact value of the gains. Just show it’s possible to find the gains so that the closed-loop system is GAS.)