AMS212B: Midterm 2

May 20, 2016

Please hand in your solutions by Monday 10AM. Points will be deducted for messy or illegible answers. You MUST work on these problems on your own, with no help from any living thing. Any suspicion of cheating by any student in the class will result in serious consequence for everyone. You may, on the other hand, use the textbook, your notes, calculators, computers, etc.

Problem 1: Find the 1-term composite expansion of the solution of this equation
\[ \epsilon \frac{d^2 f}{dx^2} - \frac{df}{dx} + (f - 1)^2 = 0 \] (1)
with boundary conditions \( f(0) = 0 \), and \( f(1) = 0 \), in the limit of small \( \epsilon > 0 \). Compare your analytical solution to a numerical solution for \( \epsilon = 0.01 \).

Problem 2: Find the 1-term composite expansion of the solution of this equation
\[ \epsilon \frac{d^2 f}{dx^2} + x \frac{df}{dx} - x^2 f = 0 \] (2)
with boundary conditions \( f(-1) = 1 \), and \( f(1) = 2 \), in the limit of small \( \epsilon > 0 \). Compare your analytical solution to a numerical solution for \( \epsilon = 0.0001 \).

Problem 3: Find 2 Frobenius Series solutions for the equation
\[ \frac{d}{dx} \left[ (1 - x^2) \frac{df}{dx} \right] + n(n+1)f = 0 \] (3)
for arbitrary \( n \) (Hint: in this problem, first expand the equation, but then leave it as is: do not divide by \( 1 - x^2 \)). Write down the first 4 terms of each solution for \( n = 1 \), \( n = 2 \) and \( n = 3 \). What do you notice, what does this mean for the solutions of this equation?

Problem 4: What is the asymptotic expansion of the solutions of the following equation, for large times \( t \):
\[ \frac{d^2 f}{dt^2} - t^4 f = 0 \] (4)
Make sure you expand the solution until you are certain the remaining terms all decay as \( t \to \infty \).