Hi, I'm working on a math problem.

Problem: Consider a random sample from a population with unknown mean \( \mu \) and variance \( \sigma^2 \).

1. Is the sample mean \( \bar{X} \) an unbiased estimator of \( \mu \)?
2. Is the sample variance \( S^2 \) an unbiased estimator of \( \sigma^2 \)?

I'm thinking about applying the Central Limit Theorem because the sample size is large.

So far, I have:

- Sample mean: \( \bar{X} \)
- Sample variance: \( S^2 \)

Please provide guidance on how to proceed.

Thank you.
map easier? IID

say is likely to be a bit more informative than IID

if $N \gg n$ \[ SRS \equiv IID \]

is a lot bigger than

under IID $Y_i = \begin{cases} 1 & \text{if homeless} \\ 0 & \text{else} \end{cases}$

under the story

\[ P(Y_i = 1 \mid B) = \theta \]

\[ P(Y_i = 0 \mid B^c) = 1 - \theta \]
\[
P(Y_i = y_i | \theta) = \begin{cases} 0 & \text{if } y_i = 1 \\ 1 - \theta & \text{else} \\ \theta \end{cases}
\]

\[
P(Y_i = y_i | \theta) = \theta (1-\theta) \quad \text{for } \mathcal{Y} = (0, 1)
\]

This is the Bernoulli(\theta) distribution.

\text{freq: \theta fixed, unknown constant}

\text{equivalent data: realization of \theta iid, i.e., unknown parameter}

\text{Bayesian notation is distributed product indexed by (unknown) parameter \theta}

\text{or (0, 1)}
Frequentist story for inference

\[ \hat{\theta} \sim \text{distribution} \]

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\[ \text{mean } \theta = \bar{\theta} \]

\[ \text{active: } \bar{\theta} = 0.0 \]

\[ \tilde{\theta} = 0.0 \]

\[ \bar{\theta} = 0.0 \]

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\[ \bar{\theta} = 0.0 \]
$Y_i \sim N(\mu, \sigma^2)$

\begin{align*}
\eta &= (\theta, \phi) = (\mu, \sigma) \\
&\text{(mean, variance)} \\
\text{variance:} &\quad \sigma^2
\end{align*}

interest

focuses on

$\mathbf{\theta} = (\mu, \sigma)$ (nuisance parameters)

$\phi = (\phi_1, \ldots, \phi_s)$