def. (conditional probability)

\[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} = BA \]

\[ P(B) = \frac{\text{Box} \setminus \Box B}{\text{Box}} = 1 \]

\[ P(B \mid A) = \begin{cases} \frac{P(A \cap B)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & \text{if } P(A) = 0 \end{cases} \]

and

\[ P(AB) = P(A) \cdot P(B \mid A) \]
If \( A \) \& \( B \) independent,

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

Frequentist def. \textit{of indy.}

Bayesian def: \( A, B \) are indep. iff information about truth of one of \( A, B \) doesn't change prob. that the other is true.

deterministic causality:

\textit{I cause if} \( A \) to occur, \textit{the effect is always} \( B \).
probabilistic causality:

if I cause A to occur, the probability of the effect B occurring goes up or down

\[ \frac{P(\text{effect} \mid \text{cause})}{P(\text{cause} \mid \text{effect})} \]

\[ P(B \mid A) \]

\[ P(A \mid B) \text{ equal only if } P(A) = P(B) \]

want to be able to go from \( P(\text{effect} \mid \text{cause}) \) to \( P(\text{cause} \mid \text{effect}) \)
how can you compute $P(B|A)$ from $P(A|B)$?

$P(B|A) = \frac{P(AB)}{P(A)}$

$P(A|B) = \frac{P(AB)}{P(B)}$

$P(AB) = P(B|A)P(A)$

$P(B) = P(B|A)P(A) + P(B|\neg A)P(\neg A)$

Bayes' Theorem for prop. $A$, $B$

Updating your uncertain (uncertainty) about truth of $B$ given info. on truth of $A$
\[
P(\text{cause} | \text{effect}) \cdot \text{data} \\
\frac{1}{P(\text{cause}) P(\text{effect} | \text{cause})} \\
\text{unknown} \\
P(\text{unknown} | \text{data}) \\
\frac{P(\text{unknown}) P(\text{data} | \text{unknown})}{P(\text{data})} \\
\text{test+} \overset{\circ}{\rightarrow} \text{test+} \overset{\circ}{\rightarrow} \text{test+} \\
\text{says HIV-} \rightarrow \text{says HIV+} \\
\text{strength of antibodies against HIV virus}
\[
P(A|B) = \frac{P(\text{ELISA} \text{ really } \text{HIV+} | \text{HIV+})}{P(\text{ELISA} \text{ really } \text{HIV+} | \text{HIV+})}\]

\[
P(\overline{A} | \overline{B}) = \frac{P(\text{ELISA} \text{ really } \text{HIV-} | \text{HIV-})}{P(\text{ELISA} \text{ really } \text{HIV-} | \text{HIV-})}
\]
\( P(\text{BIA}) \)
\( P(\text{BIA|A}) \)

Bayes’ Theorem:

\[ P(A|B) = \frac{P(B|A)P(A)}{P(B)} \]

in patient similar to this patient

1. HIV
2. A
3. B

HIV background prevalence

\( P(\text{BIA}) \rightarrow P(\text{BIA|A}) \)
\[
\text{if } P(B) - P \neq 1 \text{ then agree}
\]

\[
\text{else}
\]

\[
\text{call } \frac{P(B)}{P(\bar{B})} = \frac{P}{1-P}
\]

\[
\text{odds in favor of } B
\]

<table>
<thead>
<tr>
<th>(P)</th>
<th>(\frac{P}{1-P})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1/0.9 = 0.111</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25/0.75 = 0.33</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>0.75</td>
<td>0.75/0.25 = 3</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9/0.1 = 9</td>
</tr>
</tbody>
</table>

\[
\text{to 1 odds in favor}
\]

\[
\text{to 1 odds in favor}
\]
\[ \frac{P(BIA)}{P(BIA)} = \frac{P(B)}{P(B)} \cdot \frac{P(A|B)}{P(A|\overline{B})} \]

\[ \text{posterior} \cdot \frac{\text{prior odds of } B}{\text{likelihood ratio}} \]

\[ B = \text{really is HIV+} \]
\[ A = \text{ELISA says HIV+} \]

\[ \frac{P(B|A)}{P(B|A)} = \left( \frac{0.01}{0.99} \right) \left( \frac{0.95}{1 - 0.98} \right) \]

\[ 47.5 \text{ to } 1 \]

\[ \frac{47.5}{99} \]

\[ 99 \text{ to } 1 \text{ (prior odds in favor of HIV)} \]
\[ P(A \text{ and } B) = \frac{95}{1000} \]

\[ P(A \mid B) = \frac{95}{100} \]

\[ P(B \mid A) = \frac{95}{293} \approx 0.32 \]

\[ P(A) = \frac{293}{10000} = \frac{95 + 198}{10000} \]

\[ = \frac{95}{10000} + \frac{198}{10000} \]

\[ \text{Law of Total Probability} \]

\[ = P(A \text{ and } B) + P(A \text{ and } \overline{B}) \]
ELISA's false rate was 

\[ P(\text{HIV}^- | \text{ELISA}^+) \]

\[ = \frac{45}{293} = 0.15 \]

You are at a very low risk of infection from this blood bank.

False negatives are even worse than false positives.

You know you are safe if you test negative for HIV on every pool unit of blood.