Required Problems

1. Read the first paragraph of problem 2.7.3 (p. 112) from the text. Skip the definition of the exponential, and instead use the parameterization from class: \( f(y|\lambda) = \lambda e^{-\lambda y} \) for \( y > 0 \) and 0 otherwise (for \( \lambda > 0 \)).

   (a) Suppose you observe data \( y_1, \ldots, y_n \). Show that the gamma distribution is conjugate for \( \lambda \).

   (b) Show that the posterior mean is a weighted average of the prior mean and the data estimate (which is the reciprocal of the data mean here, and is the MLE), and find the effective sample size of the prior.

   (c) The guy from EEE has prior information from another experiment that he judges to be comparable to this one: from this other experiment the prior for \( \lambda \) should have a mean of about \( \mu = 1/4500 \) and a variance of about \( \sigma^2 = 1/11250 \) (sorry I forgot in the earlier version of this problem that when I copied these numbers from the text, I had to convert them from the other exponential parameterization).

   i. Find the gamma prior to which this information corresponds.

   ii. Use the data from the beginning of the problem (p. 112) and find the resulting posterior distribution for \( \lambda \).

   iii. Plot the prior, likelihood, and posterior on the same graph, and summarize what all of this has to say about the failure times of the metal wire samples with which the problem began.

2. Problem 2.7.5 (pp. 118–120). Just read through parts (a) and (b), as you should have them in your class notes. Do parts (c) through (f). In part (c), note that (12) should be (2.132).

Optional Problem

3. Problem 2.7.1 (pp. 110–112). Note that in part (b), condition (3) should be (2.112), and density (4) should be (2.113), although also note that in class (and in problem 2 above) the exponential density is parameterized as \( f(y|\lambda) = \lambda e^{-\lambda y} \), so you may use either parameterization here.