Note on the use of R: only include the most relevant bits of R output in your write-up. Please just cut and paste (electronically or with scissors and tape). Do not include your entire R transcript!

Required Problems

1. Do problem 2.7.2 part (a) (p. 112) from the text. The part labeled “extra credit” should be considered the optional problem for this week.

2. Consider the case of flipping a coin 100 times and getting 56 heads. Denote by \( \theta \) the probability that the coin shows heads.

   (a) Taking a frequentist approach, is it reasonable that this is a fair coin? The standard hypothesis testing approach would compute the \( p \)-value for testing the null of \( \theta = .5 \) vs. \( \theta \neq .5 \), which is the conditional probability of getting at least 56 heads given that \( \theta = .5 \) (technically it is twice this probability, since it is a two-sided test, so it should be “as or more extreme than 56 heads given the null is true”, but let’s keep things simple here by just looking at one tail).

   i. Use R to find the exact probability of getting at least 56 heads on 100 flips of a fair coin. [Just show the R command and the result.]

   ii. Use the normal approximation to the binomial (via the central limit theorem) (also use the continuity correction if you are familiar with it) to compute the approximate probability of getting at least 56 heads on 100 flips of a fair coin. [Show your work in computing the normal approximation, then use R instead of looking up some number in a normal table.]

   (b) Now take a Bayesian approach. Suppose you have a uniform prior for \( \theta \).

      i. Compute your posterior distribution for \( \theta \).

      ii. What is the posterior probability that \( \theta > .5 \)? [use R, since you should have a Beta distribution]

      iii. Compute your posterior predictive distribution for a new coin flip [the easy way to do this is to compute \( P(Y = 1 | X = 56) \), where \( Y \) is the new flip, and then note that \( P(Y = 0 | X = 56) = 1 - P(Y = 1 | X = 56) \)].

      iv. Under your posterior predictive, what is the probability of getting at least 56 heads on 100 flips of this coin? (Use R to get the probability a binomial will be at least 56 when \( n = 100 \) and \( p \) is the value you got from the previous part.)

   (c) An alternative approach is to consider interval estimates.

      i. Compute the standard frequentist confidence interval for \( \theta \), i.e., \( \hat{\theta} \pm 1.96 \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} \). Interpret this interval.

      ii. Compute an equal-tailed 95% posterior interval for \( \theta \) (since the posterior is a Beta, get the .025 and .975 quantiles of this beta distribution from R). Interpret this interval.

      iii. Explain intuitively why these intervals have slightly different values.

   (d) What do you conclude about this coin?
3. Elicit your personal prior on the probability that Schwarzenegger is re-elected governor later this year. Use the Beta family to find an appropriate distribution to approximate your beliefs. [You may want to do this by trial and error by making plots in R.] Note that Schwarzenegger has not been doing terribly well in the polls, so your point estimate is likely going to be smaller than .5, and your full distribution is unlikely to be uniform. You will have to decide exactly what it is. For this question, state the specific Beta distribution you choose, and plot its density with R. [Only include the plot in your homework, not any of the other R commands for this problem.]