AMS-205A  FALL 2009
MIDTERM

(1) [30 pts] A machine that produces electronic boards fails during a day a random number of times that follows a Poisson distribution with mean \( \lambda \). However, not all of the failures result in a defective board. Every time the machine fails, the probability that the resulting board is defective is independent from previous events and equal to \( p \). Let \( N \) be the number of defective boards produced in one day.
   (a) What is the distribution of \( N \)?
   (b) What is the expectation and variance of \( N \)?

(2) [25 pts] Forty-eight measurements are recorded to several decimal places. Each of these 48 numbers is rounded off to the nearest integer. The sum of the original 48 numbers is approximated by the sum of these integers. If we assume that the errors made by rounding off are iid and have a uniform distribution over the interval \((-1/2, 1/2)\), compute approximately the probability that the sum of the integers is within two units of the true sum.

(3) [45 pts] Let \( X_1, \ldots, X_n \) be a random sample such that \( X_i \sim \text{Exp}(\lambda) \), i.e., \( E(X_i) = \lambda \). Consider a statistic \( Y = \min_{1 \leq i \leq n} \{X_i\} = X_{(1)} \)
   (a) Find the probability density function for \( Y \). Can you recognize it as some known distribution?
   (b) Find a constant \( c \) (possible dependent of the sample size \( n \)) such that the estimator \( \hat{\lambda} = cY \) is unbiased.
   (c) Is the resulting estimator consistent? Would you use this estimator to construct a confidence interval for \( \lambda \)? Justify your answer.
   (d) Suppose that you are interested in estimating \( \theta = \lambda^2 \). Would \( \hat{\theta} = c^2 Y^2 \) be an unbiased estimator for \( \theta \)?
   (e) Consider now the estimator \( \tilde{\lambda} = \bar{X} \). Is it an unbiased estimator? Is it consistent? Create a 95\% confidence interval for \( \lambda \) based on \( \tilde{\lambda} \).